19.1 The Schwarzschild radius is \( r_s = 2M \), where \( M \) is the mass. Assuming gravity to pack neutrons down to a hardcore radius \( r_0 \simeq 0.5 \times 10^{-13} \) cm, the radius of the neutron star will be \( R \simeq r_0 A^{1/3} \), where \( A \) is the number of neutrons in the entire star (related to the total mass by \( M \simeq A m \), where \( m = 939 \) MeV \( = 1.2 \times 10^{-52} \) cm is the neutron mass).

Equating the neutron star radius to the Schwarzschild radius implies that 
\[
  r_0 A^{1/3} = 2M, \\
  R = r_0 A^{1/3} \\
\]
where \( A \) is the number of neutrons in the entire star (related to the total mass by \( M \simeq A m \), where \( m = 939 \) MeV \( = 1.2 \times 10^{-52} \) cm is the neutron mass).

By these simple considerations we estimate that the radius is 
\[
  R \approx r_0 A^{1/3} \approx 7 \text{ km}, \\
  M = R / 2 = 3.5 \text{ km} = 2.3M_\odot, \\
  \bar{\rho} = M / 4\pi R^3 \approx 0.0024 \text{ km}^{-2} = 3.2 \times 10^{15} \text{ g cm}^{-3}, \\
\]
which is larger than nuclear matter density (about \( 2.5 \times 10^{14} \) g cm\(^{-3} \)). More realistic estimates for actual neutron stars give similar numbers: a radius of 10 km, a mass of about \( 2M_\odot \), and an average density of \( 10^{15} \) g cm\(^{-3} \).

19.3 Neglecting the pressure term in the numerator and \( 2m \) relative to \( r \) in the denominator (weak gravity assumption) in Eq. (19.14), we obtain 
\[
  d\sigma / dr \sim m / r^2 \\
\]
\( 2\sigma \equiv \sigma \). But this means that \( \phi \) is just the Newtonian gravitational potential in \( G = 1 \) units (see §17.5.1). Thus, \( \sigma \) is proportional to the gravitational potential in the Newtonian limit. This interpretation is strengthened by substituting \( \phi = \frac{1}{2} \sigma \) and neglecting \( P \) relative to \( \rho \) in Eq. (19.8) to give 
\[
  dP / dr = -\frac{1}{2}(P + \rho) d\sigma / dr = -(P + \rho) d\phi / dr \sim -\rho d\phi / dr = -Gm\rho / r^2, \\
\]
where the last step follows from substituting \( d\phi / dr \sim Gm / r^2 \). This is the pressure-balance equation in Newtonian hydrostatics for a gravitational potential \( \phi = \frac{1}{2} \sigma \) (see Table 4.1).

19.5 From the solution of Exercise 19.3 the radial metric component for the Oppenheimer–Volkov solution is given by 
\[
  g_{11}(r) = \left(1 - \frac{2M(r)}{r}\right)^{-1}, \\
\]
which is unity at the center where \( M(r) = 0 \). Thus the ratio of \( g_{11} \) at the surface to that at the center is 
\[
  g_{11}(R) / g_{11}(0) = \left(1 - \frac{2M(R)}{R}\right)^{-1} \approx 2.5, \\
\]
where we’ve assumed a neutron star of radius 10 km and mass \( M = 2M_\odot \approx 3 \) km. Since the average spacing between neutrons is \( \sim 10^{-13} \) cm, this means that the metric changes by
only of order one part in $10^{19}$ over the internucleonic spacing. Thus, on that distance scale the metric is extremely flat. See Glendenning [95], §4.4 for further discussion.

19.7 Outside a spherical neutron star the metric should be well approximated by the Schwarzschild form. In the Schwarzschild metric the escape velocity is $v_{\text{esc}} = (2M/R)^{1/2}$. The exact relationship between the mass and radius of a neutron star depends on the equation of state but if $M = 1M_\odot = (2.95/2)$ km and $R = 12$ km we obtain $v_{\text{esc}}/c = 0.5$, while if $M = 2M_\odot$ and $R = 10$ km then $v_{\text{esc}}/c = 0.77$. Thus, any realistic choice of $M$ and $R$ for a neutron star will give an escape velocity that is a significant fraction of the speed of light. This is a signal that the gravitational field is very strong and general relativistic effects are significant. On the other hand, if we assume for a white dwarf that $M = 1M_\odot$ and $R = 5000$ km, the escape velocity is $v_{\text{esc}}/c \simeq 0.024$ and general relativistic effects for a white dwarf are small (but not completely negligible).