CHAPTER 10: Stellar Pulsations and Variability

10.1 The adiabatic sound speed is \( v_s = (\gamma P/\rho)^{1/2} \), where \( P \) is pressure, \( \rho \) is density, and \( \gamma \) is the adiabatic index. At hydrostatic equilibrium for constant density \( dP/dr = -\frac{4}{3} \pi G \rho^2 r \), since the mass \( m \) contained within the radius \( r \) is \( \frac{4}{3} \pi r^3 \rho \). Integrate this to give the pressure at radius \( r \) (see Exercise 4.11),

\[
P(r) = \frac{2}{3} \pi G \rho^2 (R^2 - r^2),
\]

where \( R \) is the surface radius and \( P(R) = 0 \) was assumed. Insert this expression for \( P \) in the equation for \( v_s \) and estimate the period as \( \Pi \simeq \int_0^R dr/v_s \) to give

\[
\Pi \simeq \sqrt{\frac{3 \pi}{2G \rho}}.
\]

Note that this is just another variation on estimating a hydrodynamic timescale for a star; see § 4.5 and Exercise 4.1.

10.4 In Lagrangian coordinates the equation of motion including possible deviations from hydrostatic equilibrium is obtained by rewriting Eq. (4.13) with mass as the independent variable (see § 4.3):

\[
\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2} \frac{\partial^2 r}{\partial t^2}.
\]

Assume a small time-dependent variation in \( r \) and \( P \) around the equilibrium values,

\[
r(m,t) = r_0(m)(1 + \delta r(m,t)) \quad P(m,t) = P_0(m)(1 + \delta P(m,t)),
\]

insert into the previous differential equation, expand all factors containing \( 1 + \delta P \) and \( 1 + \delta r \) to first order in a binomial expansion, and neglect resulting terms of higher than linear order in \( \delta r \) and \( \delta P \) to give the desired result.