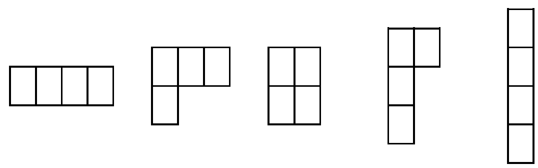
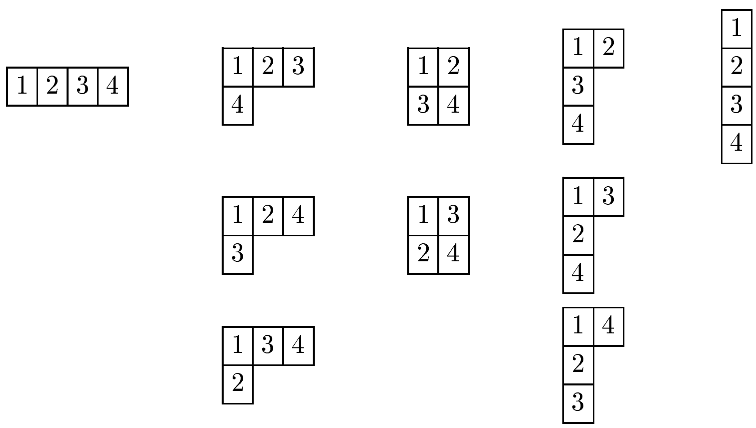
Chapter 3: Solutions to Odd-Numbered Exercises

3.1 Allowed diagrams are

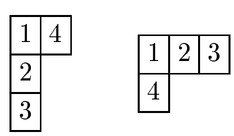


The irreps of S_n are found by enumerating n-particle diagrams with n states available, and with each particle in a different state. The possibilities are



1-D 3-D 2-D 3-D 1-D

3.3 The two diagrams are



For the first diagram, labeling the four available states by a, b, c, and d,

$$\psi_1 = \mathcal{A}_{14}\mathcal{S}_{123}abcd,$$

where $\mathcal{A}_{14}$ is the antisymmetrizing operator between particles 1 and 4, and $\mathcal{S}_{123}$ is the symmetrizing operator among particles 1, 2, and 3:

$$\begin{aligned} \mathcal{A}_{14} &= 1 - (14) \\ \mathcal{S}_{123} &= 1 + (12) + (13) + (23) + (123) + (321). \end{aligned}$$

Therefore, the wavefunction is

$$\begin{aligned}\psi_1 &= [1-(14)][1+(12)+(13)+(23)+(123)+(321)]abcd\\ &= [1-(14)](abcd+bacd+cbad+acbd+bcad+cabd).\end{aligned}$$

Multiplying out the terms then gives

$$\begin{aligned}\psi_1 = \ & abcd+bacd+cbad+acbd+bcad+cabd\\ & -dbca-dacb-dbac-dcba-dcab-dabc.\end{aligned}$$

A similar treatment of the second diagram gives

$$\begin{aligned}\psi_2 = \ & abcd-bacd-cbad-acbd+bcad+cabd\\ & +dbca-bdca-cbda-dcba+bcda+cdba.\end{aligned}$$