Physics 541
Spring, 2024
Dr. Guidry
Test 2

Do all problems. Points for each problem given in parentheses.

1. A point charge q is located in free space a distance d from the center of a dielectric sphere of radius a, with $a < d$. Find the potential at all points in space as an expansion in spherical harmonics with expansion coefficients evaluated.

2. A parallel-plate capacitor has plates of area A separated by a distance d and the plates are charged to a potential difference V using a battery.

(a) With the charging battery *disconnected* a dielectric sheet that has exactly the same width and length as a plate is inserted between the plates. Find the work done on the dielectric sheet; is it pulled in or must it be pushed in?

(b) Repeat the experiment and analysis of part (a), but with the charging battery *connected* to the plates. (25)

3. A localized current distribution of cylindrical symmetry has a current flowing only in the azimuthal direction, with a current density $\boldsymbol{J} = J(r,\theta)\hat{\boldsymbol{\phi}}$. The current is zero both outside the cylinder and near the origin. Working in Coulomb gauge, show that the corresponding vector potential $\boldsymbol{A}$ has only an azimuthal component with

$$A_\phi^{\text{in}}(r,\theta) = -\frac{\mu_0}{4\pi}\sum_l m_l r^l P_l^1(\cos\theta),$$

where $P_l^1(\cos\theta)$ is an associated Legendre polynomial and the multiple moments are given by

$$m_l = \begin{cases} -\dfrac{1}{l(l+1)}\displaystyle\int r^{-l-1}P_l^1(\cos\theta)J(r,\theta)\,d^3x & (\text{ interior}), \\ -\dfrac{1}{l(l+1)}\displaystyle\int r^{l}P_l^1(\cos\theta)J(r,\theta)\,d^3x & (\text{exterior}), \end{cases}$$

for the interior and exterior solutions, respectively. (25)

4. It was asserted in Eq. (5.29) that the vector potential in Coulomb gauge is given by

$$\boldsymbol{A}(\boldsymbol{x}) = \frac{\mu_0}{4\pi}\int \frac{\boldsymbol{J}(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}\,d^3x',$$

provided that the current distribution $\boldsymbol{J}(\boldsymbol{x}')$ vanishes sufficiently rapidly at infinity. Prove that this solution does indeed satisfy the Coulomb gauge condition. (25)