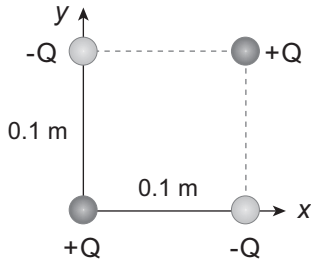
Physics 541
Spring, 2024
Dr. Guidry
Test 1

Do all problems. Points for each problem given in parentheses.

1. For the following charge distribution,



calculate the nine *cartesian components* $Q_{ij} = Q_{xx}, Q_{xy} \ldots$ of the quadruple moment tensor starting from

$$Q_{ij} \equiv \int (3x'_i x'_j - r'^2 \delta_{ij})\, \rho(\boldsymbol{x}') d^3x',$$

where $r'^2 \equiv |\boldsymbol{x}'|^2$, assuming that the origin of the coordinate system to be at the lower left charge and that $|Q| = 3\,\mu$C. (25)

2. A surface charge density specified by a function $\sigma(\theta)$ is pasted onto an empty 3D spherical shell of radius R. Assume polar coordinates with axial symmetry (r, θ) and use separation of variables in the Laplace equation to derive a general formula for the potential Φ inside and outside the shell radius R.

Hint: Remember that there is a discontinuity in the gradient of the potential when crossing a charge layer. (25)

3. A point dipole with dipole moment $\boldsymbol{p}$ is located at $\boldsymbol{x}_0$. Show that for calculations of the potential Φ or energy density W of a dipole in an external field, the dipole can be described by an effective charge density

$$\rho_{\text{eff}}(\boldsymbol{x}) = -\boldsymbol{p} \cdot \boldsymbol{\nabla} \delta(\boldsymbol{x} - \boldsymbol{x}_0),$$

where $\delta(\boldsymbol{x} - \boldsymbol{x}')$ is the 3D Dirac delta function. (25)

4. Components of the rank-2 quadrupole tensor are given by Eq. (3.117) as

$$Q_{ij} \equiv \int (3x'_i x'_j - r'^2 \delta_{ij})\, \rho(\boldsymbol{x}') d^3x'.$$

Write an expression for Q_{ij} for a discrete set of N static charges q_i and prove that it is traceless. (25)