6 Magnetic Fields in Matter

6.1 The vector potential inside the sphere is

$$A_\phi^{\text{in}}(\boldsymbol{x}) = \frac{\mu_0}{3} M_0 r \sin\theta,$$

and the magnetic field $\boldsymbol{B}$ is given by the curl of $\boldsymbol{A}$ in spherical coordinates, with the only non-vanishing terms being

$$\boldsymbol{\nabla} \times \boldsymbol{A}_\phi^{\text{in}} = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta A_\phi^{\text{in}}\right)\hat{\boldsymbol{r}} - \frac{1}{r}\frac{\partial}{\partial r}\left(rA_\phi^{\text{in}}\right)\hat{\boldsymbol{\theta}},$$

where Eq. (A.49) was used. But the second term is zero, since (why??), so letting $C = (\mu_0/3)M_0 r$,

$$\begin{aligned}
\boldsymbol{B}_{\text{in}} = \boldsymbol{\nabla} \times \boldsymbol{A}_\phi^{\text{in}} &= \frac{C}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin^2\theta\right)\hat{\boldsymbol{r}} \\
&= \frac{C}{r\sin\theta}(2\sin\theta\cos\theta)\hat{\boldsymbol{r}} \\
&= \frac{2\mu_0}{3} M_0 \cos\theta\,\hat{\boldsymbol{r}} \\
&= \frac{2\mu_0}{3} M_0 \hat{\boldsymbol{z}} \\
&= \frac{2\mu_0}{3}\boldsymbol{M},
\end{aligned}$$

and $\boldsymbol{H}_{\text{in}}$ is given by

$$\boldsymbol{H}_{\text{in}} = \frac{1}{\mu_0}\boldsymbol{B}_{\text{in}} - \boldsymbol{M} = \left(\frac{2}{3}\boldsymbol{M} - \boldsymbol{M}\right) = -\frac{1}{3}\boldsymbol{M}.$$

Thus the fields in the interior are

$$\boldsymbol{H}_{\text{in}} = -\frac{1}{3}\boldsymbol{M} \qquad \boldsymbol{B}_{\text{in}} = \frac{2\mu_0}{3}\boldsymbol{M},$$

which is the same result obtained by a different method in Eq. (6.34).