THERMONUCLEAR REACTION RATES[1]

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The changing of Bodies into Light, and Light into Bodies, is very conformable to the Course of Nature, which seems delighted with Transmutations.

Sir Isaac Newton, Knt. (1704)

Introduction

Modern computer technology has revolutionized theoretical calculations on stellar structure and stellar evolution. Physical information of great complexity, previously ignored or crudely treated, can now be introduced in detail into stellar computations and can be treated with great precision and completeness. In particular it is no longer necessary (Iben 1965, Wagoner, Fowler & Hoyle 1967) to approximate hard-won laboratory results on nuclear reaction rates by crude power-law dependences on temperature.

In this article we review the available experimental data on cross sections for the nuclear interactions of neutrons, protons, and alpha particles with a number of light and intermediate-mass nuclei and we present calculations on the resulting reaction rates, nuclear lifetimes, and energy generation rates under astrophysical conditions. We restrict our considerations to nondegenerate, nonrelativistic circumstances for the interacting nuclei. Table I lists the general nuclear processes discussed, along with the units used in presenting numerical results and the notation used in representing nuclear reactions. Previous reviews of nuclear reaction rates, somewhat more limited in scope, have been given by Fowler (1954, 1960), Burbidge, Burbidge, Fowler & Hoyle (1957) (hereafter referred to as B^2FH), Caughlan & Fowler (1962), and Parker, Bahcall & Fowler (1964). The last previous overall survey of nuclear reaction rates which has come to our attention is that of Reeves (1965).

A primary motivation in the preparation of this article has been the desire to extend where possible the calculated reaction rates to quite high temperatures of the order of 10 billion °K corresponding to interaction energies of the order of 1 MeV.[4] This extension is clearly required if applications to the ad-

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[4] It is for this reason that we have chosen 10^9 °K as the unit of temperature and 1 MeV as the unit of energy in the numerical results given in Tables II to VI. Similar tables employing 10^6 °K and keV can be obtained from the authors on request. We of course use the erg in expressing energy generation rates in erg g^{-1} sec^{-1}.

TABLE I

NUCLEAR PROCESSES DISCUSSED

Hydrogen burning
Proton-proton chain
CNO bi-cycle
Helium burning
$3He^4 \rightarrow C^{12}$, $C^{12}(\alpha,\gamma)O^{16}$, $O^{16}(\alpha,\gamma)Ne^{20}$, $Ne^{20}(\alpha,\gamma)Mg^{24}$
Silicon burning
Photodisintegration and radiative-capture rates for S^{32}, Si^{28}, Mg^{24}, etc.
Interactions of neutrons, protons, and alpha particles with light nuclei

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Units
cm, g, sec; barn $=10^{-24}$ cm^2
10^9 °K, erg, MeV $=1.6021\times10^{-6}$ erg

Nuclear notation for incident particles and the lighter products in reactions:
n, p, d, t, τ, α, Li^6, Li^7, Be^9, etc., etc.
Atomic notation for target particles and the heavier products in reactions; also used in referring to mass fractions:
H or H^1, D or D^2, T or T^3, He^3, He^4, Li^6, Li^7, Be^9, etc., etc. (Nuclear and atomic notations are identical for $A \geq 5$.)

Nondegenerate, nonrelativistic conditions for nuclei

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vanced stages of stellar evolution, giants, supergiants, and supernovae are to be made. Furthermore there has recently been much interest once again in nuclear processes during the early stages of the expanding universe (Wagoner, Fowler & Hoyle 1967) and in the implosion-explosion of supermassive stars (Fowler 1966).

In order to make extensions to high temperature, we have reanalyzed much of the experimental data in the literature and have fitted the data with semi-empirical parameters frequently different from those given by the experimental investigators. At high temperatures the nuclear products of a given reaction react to reverse the original reaction. This is particularly true near the end of burning of nuclear fuels. Photodisintegration rates become comparable to those for radiative capture. Endoergic reactions replace exoergic ones and energy is absorbed in nuclear breakup rather than released in nuclear synthesis. Thus we make extensive use of the reciprocity theorem for nuclear reactions with detailed attention to statistical and identical-particle factors and we present rates in both directions for all reactions discussed.

There is one important aspect of nuclear reaction rates which we have not treated. This is the enhancement of the rates by the screening effects of electrons surrounding the interacting nuclei. The calculation of screening factors has been treated very comprehensively by Salpeter (1954) and Wolf (1965) and has recently been reviewed by Reeves (1965). *Nor have we considered the possible modifications of reaction rates due to inelastic collisions between the compound nucleus and other particles of the gas.* Clayton & Shaw (1967) have shown that these processes become relevant at high temperature and density. Finally we have abstained from any analysis requiring the choice of nuclear interaction radii. This has seriously limited the theoretical approaches available to us but, as a result, our numerical values reflect as closely as possible the experimental data obtained in the laboratory.

Nuclear Reaction Rates Under Astrophysical Conditions

Interaction rates and mean lifetimes.—The interaction rate between two nuclei, 0 and 1, with number densities n_0 cm^{-3} and n_1 cm^{-3} is

$$P_{01} = \frac{n_0 n_1}{1 + \delta_{01}} \langle 01 \rangle \text{ reactions cm}^{-3} \text{ sec}^{-1} \qquad 1.$$

where

$$\langle 01 \rangle \equiv \langle \sigma v \rangle_{01} \text{ in cm}^3 \text{ sec}^{-1} \qquad 2.$$

is the product of cross section and velocity averaged over the appropriately normalized velocity distribution (see Equation 32 below), and δ_{01} is the Kronecker delta. In terms of the *atomic* abundances by mass, X_0 and X_1, one has

$$n_0 = \rho N_A \frac{X_0}{A_0}, \qquad n_1 = \rho N_A \frac{X_1}{A_1} \qquad 3.$$

where ρ = matter density in g cm^{-3}, $N_A = M_U^{-1} = 6.02252 \times 10^{23}$ g^{-1} is Avogadro's number, $M_U = 1.66043 \times 10^{-24}$ g is the atomic (and nuclear) mass unit on the C^{12}(atom) = 12 scale, $A_0 = M_0/M_U$, $A_1 = M_1/M_U$, M_0 = *atomic* mass of 0, and M_1 = *atomic* mass of 1.[5] In general, interacting nuclei are labeled such that $A_0 > A_1$. In referring to laboratory coordinates, we refer to A_1 as the moving incident nucleus and A_0 as the stationary target nucleus.

The mean lifetimes of the nuclei to the interaction are given by

$$\frac{1}{\tau_1(0)} = \lambda_1(0) = -\frac{1}{n_0}\left(\frac{dn_0}{dt}\right)_1 = -\frac{1}{X_0}\left(\frac{dX_0}{dt}\right)_1 = n_1\langle 01 \rangle = \rho N_A \frac{X_1}{A_1}\langle 01 \rangle = \frac{X_1}{A_1}[01] \text{ sec}^{-1} \qquad 4.$$

$$\frac{1}{\tau_0(1)} = \lambda_0(1) = -\frac{1}{n_1}\left(\frac{dn_1}{dt}\right)_0 = -\frac{1}{X_1}\left(\frac{dX_1}{dt}\right)_0 = n_0\langle 01 \rangle = \rho N_A \frac{X_0}{A_0}\langle 01 \rangle = \frac{X_0}{A_0}[01] \text{ sec}^{-1} \qquad 5.$$

where

$\tau_1(0)$ = mean lifetime of 0 for interaction with 1,
$\tau_0(1)$ = mean lifetime of 1 for interaction with 0,

[5] The following atomic masses should be used: $A_n = 1.008665$, $A_H = 1.007825$, $A_D = 2.014102$, $A_T = 3.016050$, $A_{He^3} = 3.016030$, $A_{He^4} = 4.002603$ rounded off to the user's taste. All others are equal to the appropriate integral mass number within an error of no more than 3 parts in 1000.

and the λ's are the corresponding interaction rates. The quantity $(dn_0/dt)_1$ designates the partial depletion rate in n_0 due to the interaction of 0 with 1. Analogous definitions can be made for the other partial depletion rates. When more than one set of reaction products results from the interaction of 0 and 1, additional items of notation must in principle be included. The explicit notation we have employed is discussed in connection with Equations 37 and 38 in what follows.

In Equations 4 and 5 we have introduced a convenient quantity

$$[01] \equiv \rho N_A \langle 01 \rangle \text{ sec}^{-1} \qquad 6.$$

in terms of which

$$P_{01} = \rho N_A \frac{X_0 X_1}{A_0 A_1} \frac{[01]}{(1 + \delta_{01})} \qquad 7.$$

Note that $\tau_1(0)$ and $\tau_0(1)$ do not contain $(1+\delta_{01})^{-1}$; when particle 0≡particle 1, two are lost in each reaction, canceling the factor $(1+\delta_{01})^{-1}=\frac{1}{2}$.

Reverse reactions.—As discussed in the Introduction, at elevated temperatures the interaction of the product nuclei in reversing the original reaction is important. With sufficient generality we consider, at most, three reaction products 2, 3, and 4, usually in order of increasing mass, $A_2 < A_3 < A_4$, but with emitted photons always taken last. The energy release in $0+1 \rightarrow 2+3+4$ is given by

$$\begin{aligned} Q &= E_{234} - E_{01} \\ &= (M_0 + M_1 - M_2 - M_3 - M_4)c^2 \\ &= 931.478(A_0 + A_1 - A_2 - A_3 - A_4) \text{ MeV} \\ &= 1.49232 \times 10^{-3}(A_0 + A_1 - A_2 - A_3 - A_4) \text{ erg} \end{aligned} \qquad 8.$$

where E_{01} and E_{234} are the center-of-momentum energies for the incident and outgoing particles respectively. The ratio Q/k appears in numerous equations and is numerically equal to 11.605 Q_6 in 10^9 °K where $Q_6 = Q$ in MeV. In the great majority of the reactions to be discussed only two nuclear reaction products emerge; in this case, set $A_4 = 0$ and $E_{234} = E_{23}$. This also holds if a photon is emitted along with two nuclei. In the case of two reaction products, one of which is a photon, set $A_3 = 0$.

It will be noted that *atomic* rather than *nuclear* masses have been used in calculating Q, in keeping with common practice in nuclear physics. Except in reactions involving positron emission, the difference involves only changes in atomic binding energies which are in general quite small. The energy release in astrophysical circumstances is complicated by the energy of the Coulomb interactions between nuclei and bound or free electrons which can be larger or smaller than ordinary atomic binding energies. In any case nuclear energy differences are no more accurate than atomic differences, so, to avoid confusion with tabulated Q values, the *atomic* values have been used. In the case of positron emission the atomic mass differences equal the nuclear differences to high approximation *plus* the energy, $2m_ec^2 = 1.0220$ MeV $= 1.6374 \times 10^{-6}$ erg, which results from the eventual annihilation of the positron with an electron. Since this is available thermonuclear energy, it should indeed be included in the effective Q value. In the case of electron emission, the atomic mass differences automatically incorporate the mass of the electron without explicit introduction into the expression for Q.

In the case of two interacting nuclei and two reaction product nuclei, the reciprocity theorem for nuclear reactions (Blatt & Weisskopf 1952), with due regard for spin statistical factors and possible particle identities, yields in the nonrelativistic approximation

$$\frac{\sigma(23)}{\sigma(01)} = \frac{(1+\delta_{23})}{(1+\delta_{01})} \frac{g_0 g_1}{g_2 g_3} \frac{A_0 A_1 E_{01}}{A_2 A_3 E_{23}} \qquad 9.$$

where $\sigma(01)$ is the cross section for $0+1 \rightarrow 2+3$ and $\sigma(23)$ is that for $2+3 \rightarrow 0+1$. Thus (Fowler & Vogl 1964)

$$\frac{[23]}{[01]} = \frac{\langle 23 \rangle}{\langle 01 \rangle} = \frac{(1+\delta_{23})}{(1+\delta_{01})} \frac{g_0 g_1}{g_2 g_3} \left(\frac{A_0 A_1}{A_2 A_3}\right)^{3/2} \exp(-Q/kT) \qquad 10.$$

where $g_I = 2J_I + 1$ and J_I is the spin of the appropriate nucleus. Where convenient we employ I to represent 0, 1, 2, 3, 4. The introduction of the $(1+\delta)$ factors reflects the fact that cross sections between identical particles are twice those between different particles, other factors being equal.

In nuclear processes at high temperatures the excited states of nuclei frequently take part. If equilibrium between the excited state and the ground state has not been attained, then the excited nucleus must be treated as a component of the overall composition in the same way as the ground-state nucleus. In most cases of interest there will be excited states in equilibrium with the ground state, even though general equilibrium has not been attained. A test can be made by comparing the lifetime $\tau(I \rightarrow I^*)$ for photoexcitation from the ground to the excited state with the time scale for the astrophysical circumstance under consideration, t. Equilibrium will have been attained if $\tau(I \rightarrow I^*) \lesssim t$. The lifetime $\tau(I \rightarrow I^*)$ can be computed from the *spontaneous* decay lifetime of the excited state, $\tau_{sp}(I^* \rightarrow I)$, which is frequently known, by using

$$\frac{\tau(I \rightarrow I^*)}{\tau_{sp}(I^* \rightarrow I)} = \frac{g_I}{g_{I^*}} [\exp(E_{I^*}/kT) - 1] \qquad 11.$$

where E_{I^*} is the excitation energy of the excited state and T is the ambient temperature. The effects of *induced* emission on $I^* \rightarrow I$ are incorporated by the inclusion of the -1 in Equation 11.

The reactions involving all excited states in equilibrium with the ground state can be included in Equation 9 if the g_I are replaced by the partition functions

$$G_I = \sum_{I^*} g_{I^*} \exp(-E_{I^*}/kT) \qquad 12.$$

where the sum over I^* includes the ground state and runs through all excited states for which $\tau(I \rightarrow I^*) \leq t$. We make the nonrelativistic approximation that $A_{I^*} \approx A_I$. The quantity $\langle 01 \rangle$ must include sums over all transitions to the relevant excited states in nuclei 2 and 3 and must be appropriately averaged over all possible products, $n_0 n_1$, $n_0^* n_1$, $n_1^* n_0$, $n_1^* n_0^*$ . . . . , of the number densities of the ground and excited states of nuclei 0 and 1 without regard to the possible identity of 0 and 1. The factor $(1+\delta_{01})^{-1}$ in Equation 9 automatically corrects for the case of identical particles. Then n_0 and n_1 in Equation 1 become the sums over all states of nucleus 0 and 1 respectively. Similar statements, appropriately modified, apply to n_2, n_3, and $\langle 23 \rangle$, and in Equation 9 to the factor $1+\delta_{23}$.

In Equation 9 the A_I now represent the *nuclear* masses on the atomic-nuclear mass scale. Problems do arise concerning which mass, *atomic* or *nuclear*, to use in equations involving *dynamical* considerations such as Equation 9 above and many of the equations to follow. The matter has been discussed in some detail by Christy (1961). Whereas we used *atomic* masses in Equation 3 for n_0 and n_1, in Equation 8 for Q, and in Equations 28–31 for $\mathcal{E}$ below, in all other cases we have elected to use *nuclear* masses which can be computed from the tabulated *atomic* masses by the use of the relation

$$A_I \text{ (nuclear)} = A_I \text{ (atomic)} - 0.5486 \times 10^{-3} Z_I + 1.68 \times 10^{-8} Z_I^{7/3} \qquad 13.$$

where Z_I is the atomic number of nucleus I. The second term on the right-hand side corrects for the atomic electron masses while the last term represents the Fermi-Thomas binding energy ($15.6\ Z_I^{7/3}$ eV.) The differences are small and we have chosen to use the same notation for *atomic* and *nuclear* masses and to treat them as equal when both appear in the same equation, as in Equations 18 and 19 below. Our calculations have been carried out with the difference taken into account.

The mean lifetimes $\tau_2(3)$ and $\tau_3(2)$ can be calculated from [23] in the same way that $\tau_1(0)$ and $\tau_0(1)$ were obtained from [01] above. The overall reaction rate for $0+1 \rightleftharpoons 2+3$ is given by

$$P_{01} - P_{23} = \frac{n_0 n_1}{1+\delta_{01}} \langle 01 \rangle - \frac{n_2 n_3}{1+\delta_{23}} \langle 23 \rangle = \frac{\rho N_A}{1+\delta_{01}} \frac{X_0 X_1}{A_0 A_1} [01] - \frac{\rho N_A}{1+\delta_{23}} \frac{X_2 X_3}{A_2 A_3} [23] \qquad 14.$$

so that at *equilibrium*, when $P_{01} = P_{23}$,

$$\frac{n_2 n_3}{n_0 n_1} = \frac{X_2 X_3}{A_2 A_3} \frac{A_0 A_1}{X_0 X_1} = \frac{g_2 g_3}{g_0 g_1} \left(\frac{A_2 A_3}{A_0 A_1} \right)^{3/2} \exp(+Q/kT) \qquad 15.$$

In the case of radiative capture, i.e., when particle 3, say, is a photon, it can be shown that the reverse photodisintegration rate for nucleus 2 is

$$\frac{1}{\tau_\gamma(2)} = \lambda_\gamma(2) = \frac{g_0 g_1}{(1+\delta_{01}) g_2} \left(\frac{A_0 A_1}{A_2} \right)^{3/2} \left(\frac{M_U kT}{2\pi\hbar^2} \right)^{3/2} \langle 01 \rangle \exp(-Q/kT) \text{ sec}^{-1}$$

$$= 0.98677 \times 10^{10} \frac{g_0 g_1}{(1+\delta_{01}) g_2} \left(\frac{A_0 A_1}{A_2} \right)^{3/2} \rho^{-1} T_9^{3/2} [01] \exp(-11.605\ Q_6/T_9) \qquad 16.$$

where $T_9 = T$ in 10^9 °K, and $Q_6 = Q$ in MeV. The overall reaction rate is given by

$$P_{01} - P_{2\gamma} = \frac{n_0 n_1}{1+\delta_{01}} \langle 01 \rangle - \frac{n_2}{\tau_\gamma(2)} = \frac{\rho N_A}{1+\delta_{01}} \frac{X_0 X_1}{A_0 A_1} [01] - \rho N_A \frac{X_2}{A_2} \lambda_\gamma(2) \qquad 17.$$

so that at equilibrium, when $P_{01} = P_{2\gamma}$,

$$\frac{n_2}{n_0 n_1} = \frac{X_2 A_0 A_1}{\rho N_A X_0 X_1 A_2} = \frac{g_2}{g_0 g_1} \left(\frac{A_2}{A_0 A_1} \right)^{3/2} \left(\frac{2\pi\hbar^2}{M_U kT} \right)^{3/2} \exp(+Q/kT) \text{ cm}^3 \qquad 18.$$

and

$$\frac{X_2}{X_0 X_1} = 1.0134 \times 10^{-10} \rho T_9^{-3/2} \left(\frac{g_2}{g_0 g_1} \right) \left(\frac{A_2}{A_0 A_1} \right)^{5/2} \exp(11.605\, Q_6/T_9) \qquad 19.$$

In the case of three nuclear reaction products, it can be shown that

$$\frac{\langle 234 \rangle}{\langle 01 \rangle} = \frac{g_0 g_1}{g_2 g_3 g_4} \left(\frac{A_0 A_1}{A_2 A_3 A_4} \right)^{3/2} \left(\frac{2\pi\hbar^2}{M_U kT} \right)^{3/2} \left(\frac{1+\Delta_{234}}{1+\delta_{01}} \right) \exp(-Q/kT) \text{ cm}^3 \qquad 20.$$

and

$$\frac{[234]}{[01]} = \rho N_A \frac{g_0 g_1}{g_2 g_3 g_4} \left(\frac{A_0 A_1}{A_2 A_3 A_4} \right)^{3/2} \left(\frac{2\pi\hbar^2}{M_U kT} \right)^{3/2} \left(\frac{1+\Delta_{234}}{1+\delta_{01}} \right) \exp(-Q/kT)$$
$$= 1.0134 \times 10^{-10} \frac{g_0 g_1}{g_2 g_3 g_4} \left(\frac{A_0 A_1}{A_2 A_3 A_4} \right)^{3/2} \left(\frac{1+\Delta_{234}}{1+\delta_{01}} \right) \rho T_9^{-3/2} \times \exp(-11.605\, Q_6/T_9) \qquad 21.$$

where

$$\Delta_{234} = \delta_{23} + \delta_{34} + \delta_{42} + 2\delta_{234} \qquad 22.$$

In the above expressions $[234] = \rho^2 N_A^2 \langle 234 \rangle$ sec^{-1} and these new quantities are defined in the equations

$$P_{234} = \frac{n_2 n_3 n_4}{1+\Delta_{234}} \langle 234 \rangle \quad \text{reactions cm}^{-3}\text{ sec}^{-1} = \rho N_A \frac{X_2 X_3 X_4}{A_2 A_3 A_4} \frac{[234]}{1+\Delta_{234}} \qquad 23.$$

The overall reaction rate is given by

$$P_{01} - P_{234} = \frac{n_0 n_1}{1+\delta_{01}} \langle 01 \rangle - \frac{n_2 n_3 n_4}{1+\Delta_{234}} \langle 234 \rangle = \frac{\rho N_A}{1+\delta_{01}} \frac{X_0 X_1}{A_0 A_1} [01] - \frac{\rho N_A}{1+\Delta_{234}} \frac{X_2 X_3 X_4}{A_2 A_3 A_4} [234] \qquad 24.$$

so that at equilibrium, when $P_{01} = P_{234}$,

$$\frac{n_2 n_3 n_4}{n_0 n_1} = \rho N_A \frac{X_2 X_3 X_4}{A_2 A_3 A_4} \frac{A_0 A_1}{X_0 X_1} = \frac{g_2 g_3 g_4}{g_0 g_1} \left(\frac{A_2 A_3 A_4}{A_0 A_1} \right)^{3/2} \left(\frac{M_U k T}{2\pi\hbar^2} \right)^{3/2} \exp(+Q/kT) \qquad 25.$$

and

$$\frac{X_2X_3X_4}{X_0X_1} = 0.98677 \times 10^{10} \rho^{-1} T_9^{3/2} \left(\frac{g_2g_3g_4}{g_0g_1}\right) \left(\frac{A_2A_3A_4}{A_0A_1}\right)^{3/2} \exp(11.605\, Q_6/T_9) \quad 26.$$

Energy generation.—The energy generation in the forward reaction $0+1 \rightarrow 2+3+4+Q$, usually *exoergic* with positive Q, is given by

$$\mathcal{E}_{01} = \frac{P_{01}}{\rho} Q(\text{ergs})$$

$$= 1.6021 \times 10^{-6} \frac{P_{01}}{\rho} Q_6 \text{ erg g}^{-1} \text{ sec}^{-1} \quad 27.$$

so that

$$\mathcal{E}_{01} = 9.6487 \times 10^{17} \frac{X_0X_1}{A_0A_1} \frac{[01]}{1+\delta_{01}} Q_6 \text{ erg g}^{-1} \text{ sec}^{-1} \quad 28.$$

Similarly for the *endoergic* reactions:

$$\mathcal{E}_{23} = -9.6487 \times 10^{17} \frac{X_2X_3}{A_2A_3} \frac{[23]}{1+\delta_{23}} Q_6 \text{ erg g}^{-1} \text{ sec}^{-1} \quad 29.$$

$$\mathcal{E}_{2\gamma} = -9.6487 \times 10^{17} \frac{X_2}{A_2} \lambda_\gamma(2)\, Q_6 \text{ erg g}^{-1} \text{ sec}^{-1} \quad 30.$$

$$\mathcal{E}_{234} = -9.6487 \times 10^{17} \frac{X_2X_3X_4}{A_2A_3A_4} \frac{[234]}{1+\Delta_{234}} Q_6 \text{ erg g}^{-1} \text{ sec}^{-1} \quad 31.$$

At high temperatures the reverse reactions must be taken into account and the overall energy generation is $\mathcal{E}_{01}+\mathcal{E}_{23}$, $\mathcal{E}_{01}+\mathcal{E}_{2\gamma}$, or $\mathcal{E}_{01}+\mathcal{E}_{234}$ as the case may be. In Tables II to VI to be discussed later, the factors $(1+\delta_{01})^{-1}$, $(1+\delta_{23})^{-1}$, and $(1+\Delta_{234})^{-1}$ have been included in the numerical coefficients for $\mathcal{E}_{01}$, $\mathcal{E}_{23}$, and $\mathcal{E}_{234}$.

In reactions in which neutrinos are produced, the energy carried away by the neutrinos must be subtracted from Q to obtain the available thermonuclear energy generation under most circumstances. It must always be borne in mind that neutrino losses from a given reaction increase with temperature since from $\sim\frac{1}{2}$ to all of the thermal energy of the interacting particles is carried away by neutrinos. Thus, for example, the neutrino losses given in footnotes a, b, and c of Table III (to be discussed later) hold only in the limit of zero temperature.

The determination of $\langle\sigma v\rangle$.—The mean product of cross section and velocity appearing in Equations 1 and 2 enters into all the calculations discussed in the previous section. In the applications under discussion in this article it can be assumed that all nuclei and radiation are in thermodynamic equilibrium under nondegenerate, nonrelativistic conditions insofar as velocity and energy distribution are concerned. For a discussion of lepton and nuclear processes under degenerate or relativistic (or both) conditions, the reader is referred to Bahcall & Wolf (1964), Wolf (1966), Fowler & Hoyle (1964), Wagoner, Fowler & Hoyle (1967), and references therein. Using the Maxwell-Boltzmann distribution in particle velocities, transforming to the center-of-momentum energy E as the integration variable, and dropping superfluous subscripts, we found for two interacting particles that

$$\langle\sigma v\rangle = \frac{(8/\pi)^{1/2}}{M^{1/2}(kT)^{3/2}} \int \sigma E \exp(-E/kT) dE$$

$$= 6.1968 \times 10^{-14} A^{-1/2} T_9^{-3/2} \int \sigma_b E_6 \exp(-11.605\, E_6/T_9) dE_6 \text{ cm}^3 \text{ sec}^{-1} \quad 32.$$

where σ_b is the cross section in barns (10^{-24} cm^2), E_6 is the energy in MeV, and $A = A_0A_1/(A_0+A_1)$, for example, is the reduced mass.

In only a few cases are sufficient experimental data available to carry out the calculation prescribed in Equation 32 over the wide range of temperatures encountered in astrophysical applications. It is customary to evaluate $\langle\sigma v\rangle$ at low temperatures using nonresonant cross sections extrapolated from measurements made in the laboratory at the lowest energies at which the reaction can be detected. Indirect measurements on the states of the compound nucleus involved must be made to ascertain whether or not resonances occur and are important at low temperatures. At higher temperatures *measured* nonresonant cross sections are relevant and in addition, resonances occur in profusion. Nonresonant cross sections constitute the slowly varying background on which resonances are superimposed. We will first present the standard treatment of resonant and nonresonant cross sections and will then make a comparison with the exact integrals of Equation 32 in two cases where this can be done.

Nonresonant cross sections, neutrons.—The energy dependence of neutron-interaction cross sections has been discussed by Fowler & Vogl (1964). At low energy the s wave ($l=0$) interactions dominate, and the cross section is proportional to the square of the De Broglie wavelength $\lambda\!\!\!^{-}$ and the partial width for neutron emission Γ_n. Since $\lambda\!\!\!^{-}$ is proportional to v^{-1} while Γ_n is proportional to v, where v is the relative velocity of the neutron and interacting nucleus, it follows that σ is proportional to v^{-1} or σv is constant. Deviations from this occur at higher energies when other partial waves become important and σ no longer depends linearly on Γ_n as this width becomes comparable to other partial widths. When significant deviations occur, it is convenient to express the nonresonant, slowly varying behavior of $\langle\sigma v\rangle$ as the first three terms of a Maclaurin series in the velocity v of the neutron or, alternatively, a similar series in the square root of the neutron energy $E^{1/2}$, in the center-of-momentum system. Thus

$$\sigma v = \mathcal{S}(E^{1/2}) = \mathcal{S}(0)\left(1 + \frac{\dot{\mathcal{S}}(0)}{\mathcal{S}(0)} E^{1/2} + \frac{1}{2}\frac{\ddot{\mathcal{S}}(0)}{\mathcal{S}(0)} E\right) \text{cm}^3 \text{ sec}^{-1} \quad 33.$$

where $\mathcal{S}(0)$, $\dot{\mathcal{S}}(0)$, and $\mathcal{S}(0)$ are empirical constants and the dot indicates differentiation with respect to $E^{1/2} \propto v$. Substitution in Equation 32 then yields

$$\langle\sigma v\rangle = \mathcal{S}(0)\left[1 + 2\pi^{-1/2}\frac{\dot{\mathcal{S}}(0)}{\mathcal{S}(0)}(kT)^{1/2} + \frac{3}{4}\frac{\ddot{\mathcal{S}}(0)}{\mathcal{S}(0)} kT\right]$$

$$= \mathcal{S}(0)\left[1 + 0.3312\frac{\dot{\mathcal{S}}(0)}{\mathcal{S}(0)} T_9^{1/2} + 0.06463\frac{\ddot{\mathcal{S}}(0)}{\mathcal{S}(0)} T_9\right] \quad 34.$$

TABLE II

NEUTRON NONRESONANT-REACTION DATA

Reaction		$H^1(n,\gamma)D^2$	$D^2(n,\gamma)T^3$	$He^3(n,\gamma)He^4$	$He^3(n,p)T^3$	$Li^6(n,\gamma)Li^7$	$Li^6(n,t)He^4$
Parameters in typical equations indicated in parentheses							
Q	(8)	2.225	6.257	20.578	0.764	7.253	4.785
$(Q/k)_9$	(39)(43)	2.582E 01	7.262E 01	2.388E 02	8.864	8.417E 01	5.553E 01
σ_{th}	(35)	3.32 E−01	5.00 E−04	5.00 E−05	5.33 E 03	4.50 E−02	9.45 E 02
		±0.02 E−01	±1.00 E−04	±5.00 E−05	±0.01 E 03	±1.00 E−02	±0.10 E 02
$\mathcal{S}(0)$	(36)	7.30 E−20	1.10 E−22	1.10 E−23	1.17 E−15	9.90 E−21	2.08 E−16
$\dot{\mathcal{S}}(0)/\mathcal{S}(0)$	(36)	−2.60	—	—	−1.80	—	—
$\frac{1}{2}\ddot{\mathcal{S}}(0)/\mathcal{S}(0)$	(36)	3.32	1.46 E 02	7.00 E 03	1.42	—	—
Coefficients in typical equations indicated in parentheses							
$T_9^{1/2}$	(38)	−8.60 E−01	—	—	−5.97 E−01	—	—
T_9	(38)	4.29 E−01	1.89 E 01	9.05 E 02	1.83 E−01	—	—
⟨01⟩	(37)	7.30 E−20	1.10 E−22	1.10 E−23	1.17 E−15	9.90 E−21	2.08 E−16
[01]	(38)	4.40 E 04	6.62 E 01	6.62	7.06 E 08	5.96 E 03	1.25 E 08
⟨23⟩	(44)	—	—	—	1.17 E−15	—	2.22 E−16
$[\lambda_\gamma]$	(39)(43)	2.07 E 14	1.08 E 12	1.73 E 11	7.07 E 08	7.10 E 13	1.34 E 08
$\tau_1(0)$	(40)	2.29 E−05	1.52 E−02	1.52 E−01	1.43 E−09	1.69 E−04	8.06 E−09
$\tau_0(1)$	(41)	2.29 E−05	3.04 E−02	4.55 E−01	4.27 E−09	1.01 E−03	4.80 E−08
$\mathcal{E}_{01}$	(42)	9.29 E 22	1.97 E 20	4.32 E 19	1.71 E 26	6.88 E 21	9.53 E 25
Uncertainty		±10%	±30%	FAC 2	±10%	±30%	±10%
T_9 limits		0 to 5	0 to 3	0 to 3	0 to 10	0 to 2	0 to 10
References		Hu 58, Be 56	St 64, Gr 63	Zu 63	St 64	St 64	Hu 58

Reaction		$Li^7(n,\gamma)Li^8$	$Be^7(n,p)Li^7$	$Be^9(n,\gamma)Be^{10}$	$B^{10}(n,\gamma)B^{11}$	$B^{10}(n,\alpha)Li^7$	$B^{11}(n,\gamma)B^{12}$
Parameters in typical equations indicated in parentheses							
Q	(8)	2.033	1.644	6.815	11.456	2.792	3.369
$(Q/k)_9$	(39)(43)	2.359E 01	1.908E 01	7.909E 01	1.329E 02	3.240E 01	3.910E 01
σ_{th}	(35)	3.70 E−02	5.10 E 04	9.50 E−03	5.00 E−01	3.84 E 03	5.00 E−03
		±0.40 E−02	±0.60 E 04	±1.00 E−03	±2.00 E−01	±0.01 E 03	±3.00 E−03
$\mathcal{S}(0)$	(36)	8.14 E−21	1.12 E−14	2.09 E−21	1.10 E−19	8.44 E−16	1.10 E−21
Coefficients in typical equations indicated in parentheses							
⟨01⟩	(37)	8.14 E−21	1.12 E−14	2.09 E−21	1.10 E−19	8.44 E−16	1.10 E−21
[01]	(38)	4.90 E 03	6.76 E 09	1.26 E 03	6.62 E 04	5.08 E 08	6.62 E 02
⟨23⟩	(44)	—	1.12 E−14	—	—	6.37 E−16	—
$[\lambda_\gamma]$	(39)(43)	6.41 E 13	6.77 E 09	8.59 E 13	2.01 E 15	3.84 E 08	1.55 E 13
$\tau_1(0)$	(40)	2.06 E−04	1.49 E−10	8.01 E−04	1.52 E−05	1.98 E−09	1.52 E−03
$\tau_0(1)$	(41)	1.43 E−03	1.04 E−09	7.16 E−03	1.51 E−04	1.97 E−08	1.66 E−02
$\mathcal{E}_{01}$	(42)	1.36 E 21	1.51 E 27	9.11 E 20	7.25 E 22	1.36 E 26	1.94 E 20
Uncertainty		±30%	±30%	±30%	±50%	±10%	FAC 2
T_9 limits		0 to 5	0 to 3	0 to 5	0 to 2	0 to 10	0 to 0.03
References		La 66	La 66	St 64	Hu 58	St 64	Aj 67

Units: Energy in MeV; σ_{th} and ± standard deviation in barns; $\mathcal{S}(0)$ in cm³ sec⁻¹, $\dot{\mathcal{S}}(0)/\mathcal{S}(0)$ in MeV$^{-1/2}$, $\frac{1}{2}\ddot{\mathcal{S}}(0)/\mathcal{S}(0)$ in MeV^{-1}; ⟨ ⟩ in cm³ sec⁻¹; [ ] in sec⁻¹; τ in sec; $\mathcal{E}_{01}$ in erg g⁻¹ sec⁻¹; $T_9 = T/10^9$.

TABLE II (*concluded*)

NEUTRON NONRESONANT-REACTION DATA

Reaction		$C^{12}(n,\gamma)C^{13}$	$C^{13}(n,\gamma)C^{14}$	$N^{14}(n,\gamma)N^{15}$	$N^{14}(n,p)C^{14}$	$N^{15}(n,\gamma)N^{16}$	$O^{16}(n,\gamma)O^{17}$
		Parameters in typical equations indicated in parentheses					
Q	(8)	4.947	8.176	10.835	0.626	2.487	4.143
$(Q/k)_9$	(39)(43)	5.741E 01	9.488E 01	1.257E 02	7.269	2.886E 01	4.807E 01
σ_{th}	(35)	3.40 E−03 ±0.30 E−03	9.00 E−04 ±2.00 E−04	7.50 E−02 ±0.80 E−02	1.81 ±0.05	2.40 E−05 ±0.80 E−05	1.78 E−04 ±0.25 E−04
$\mathcal{S}(0)$	(36)	7.48 E−22	1.98 E−22	1.65 E−20	3.98 E−19	5.28 E−24	3.92 E−23
$\frac{1}{2}\ddot{\mathcal{S}}(0)/\mathcal{S}(0)$	(36)	—	7.70 E 02*	—	—	7.70 E 02*	7.70 E 02*
		Coefficients in typical equations indicated in parentheses					
T_9	(38)	—	1.00 E 02*	—	—	1.00 E 02*	1.00 E 02
⟨01⟩	(37)	7.48 E−22	1.98 E−22	1.65 E−20	3.98 E−19	5.28 E−24	3.92 E−23
[01]	(38)	4.50 E 02	1.19 E 02	9.94 E 03	2.40 E 05	3.18	2.36 E 01
⟨23⟩	(44)	—	—	—	1.20 E−18	—	—
$[\lambda_\gamma]$	(39)(43)	3.99 E 12	4.27 E 12	2.69 E 14	7.21 E 05	2.31 E 10	7.17 E 10
$\tau_1(0)$	(40)	2.24 E−03	8.46 E−03	1.02 E−04	4.21 E−06	3.17 E−01	4.28 E−02
$\tau_0(1)$	(41)	2.66 E−02	1.09 E−01	1.41 E−03	5.84 E−05	4.72	6.78 E−01
$\mathcal{E}_{01}$	(42)	1.78 E 20	7.17 E 19	7.36 E 21	1.03 E 22	5.04 E 17	5.84 E 18
Uncertainty		±30%	FAC 2	±30%	±20%	FAC 2	FAC 2
T_9 limits		0 to 5	0 to 3	0 to 5	0 to 5	0 to 3	0 to 3
References		St 64	St 64, Fo 67	St 64	St 64	Hu 58, Fo 67	St 64, Fo 67

Reaction		$O^{17}(n,\alpha)C^{14}$	$O^{18}(n,\gamma)O^{19}$	$F^{19}(n,\gamma)F^{20}$	$Ne^{21}(n,\alpha)O^{18}$	$Ne^{22}(n,\gamma)Ne^{23}$	$Na^{23}(n,\gamma)Na^{24}$
		Parameters in typical equations indicated in parentheses					
Q	(8)	1.819	3.956	6.597	0.699	5.195	6.962
$(Q/k)_9$	(39)(43)	2.111E 01	4.591E 01	7.656E 01	8.114	6.029E 01	8.079E 01
σ_{th}	(35)	2.35 E−01 ±0.10 E−01	2.10 E−04 ±0.40 E−04	9.80 E−03 ±0.70 E−03	9.60 E 01 ±3.30 E 01	3.60 E−02 ±1.50 E−02	5.34 E−01 ±0.05 E−01
$\mathcal{S}(0)$	(36)	5.17 E−20	4.62 E−23	2.16 E−21	2.11 E−17	7.92 E−21	1.17 E−19
$\dot{\mathcal{S}}(0)/\mathcal{S}(0)$	(36)	—	—	5.40 E 03	—	—	5.00 E 01
$\frac{1}{2}\ddot{\mathcal{S}}(0)/\mathcal{S}(0)$	(36)	7.70 E 02*	7.70 E 02*	−0.97 E 04	—	—	−9.00 E 01
		Coefficients in typical equations indicated in parentheses					
$T_9^{1/2}$	(38)	—	—	1.79 E 03	—	—	1.66 E 01
T_9	(38)	1.00 E 02*	1.00 E 02*	−1.25 E 03	—	—	−1.16 E 01
⟨01⟩	(37)	5.17 E−20	4.62 E−23	2.16 E−21	2.11 E−17	7.92 E−21	1.17 E−19
[01]	(38)	3.11 E 04	2.78 E 01	1.30 E 03	1.27 E 07	4.77 E 03	7.08 E 04
⟨23⟩	(44)	1.05 E−19	—	—	2.69 E−17	—	—
$[\lambda_\gamma]$	(39)(43)	6.32 E 04	8.55 E 10	9.61 E 12	1.62 E 07	1.49 E 13	5.90 E 14
$\tau_1(0)$	(40)	3.24 E−05	3.63 E−02	7.77 E−04	7.93 E−08	2.11 E−04	1.43 E−05
$\tau_0(1)$	(41)	5.46 E−04	6.47 E−01	1.46 E−02	1.65 E−06	4.61 E−03	3.25 E−04
$\mathcal{E}_{01}$	(42)	3.19 E 21	5.85 E 18	4.31 E 20	4.05 E 23	1.08 E 21	2.05 E 22
Uncertainty		FAC 2	FAC 2	FAC 2	±50%	±50%	FAC 2
T_9 limits		0 to 3	0 to 3	0 to 2	0 to 3	0 to 2	0 to 2
References		St 64, Fo 67	Hu 58, Fo 67	St 64, Ma 65	St 64	Hu 58	St 64, Ma 65

Units: Energy in MeV; σ_{th} and ± standard deviation in barns; $\mathcal{S}(0)$ in cm³ sec⁻¹, $\dot{\mathcal{S}}(0)/\mathcal{S}(0)$ in MeV$^{-1/2}$, $\frac{1}{2}\ddot{\mathcal{S}}(0)/\mathcal{S}(0)$ in MeV^{-1}; ⟨ ⟩ in cm³ sec⁻¹; [ ] in sec⁻¹; τ in sec; $\mathcal{E}_{01}$ in erg g⁻¹ sec⁻¹; $T_9 = T/10^9$.

* Estimated.

where, for $E_6 = E$ in MeV, $\dot{\mathcal{S}}(0)/\mathcal{S}(0)$ is expressed in the unit $\text{MeV}^{-1/2}$, and $\ddot{\mathcal{S}}(0)/\mathcal{S}(0)$ in MeV^{-1}. In the great majority of cases, the available data do not warrant detailed analysis, but in a few cases, least-squares analysis of accurate, extensive data has been performed to determine $\dot{\mathcal{S}}(0)$ and $\ddot{\mathcal{S}}(0)$. In all cases $\mathcal{S}(0)$ has been predetermined from the accurately measured thermal-neutron cross section σ_{th} at $v_{th} = 2.20 \times 10^5$ cm sec^{-1} or $E_{lab} = 2.53 \times 10^{-8}$ MeV. Thus

$$\begin{aligned} \mathcal{S}(0) &= (\sigma v)_{th} \\ &= 2.20 \times 10^{-19}\, \sigma_{th}\,(\text{barn})\ \text{cm}^3\ \text{sec}^{-1} \end{aligned} \qquad 35.$$

Whenever $\mathcal{S}(0)$ is anomalously low, indicating the operation of some selection rule for the s-wave interaction, we have estimated a p-wave ($l = 1$) contribution which arbitrarily increases $\langle \sigma v \rangle$ by a factor of 100 at $T_9 = 1$. We have marked the estimates made in this way in Table II with an asterisk.

The results for a number of neutron-induced reactions are tabulated in Table II. Empirical values for $\langle \sigma v \rangle$ as a function of temperature have been given for a large number of intermediate and heavy nuclei in a comprehensive paper by Macklin & Gibbons (1965) and are not repeated here, except for a very approximate treatment by our methods of their results for $F^{19}(n,\gamma)F^{20}$ and $Na^{23}(n,\gamma)Na^{24}$. The use of Table II will be illustrated by using the reaction $H^1(n,\gamma)D^2$ as an example. For this reaction $Q = 2.225$ MeV and $Q/k = 25.82 \times 10^9$ °K. The latter number occurs in the exponential in Equation 39 below. The thermal-capture cross section is 0.332 ± 0.002 barn (Hughes & Schwartz 1958) so that $\mathcal{S}(0) = 7.30 \times 10^{-20}$ cm^3 sec^{-1}. We have used the theoretical calculations and data on the photodisintegration of deuterium, the reverse reaction, as discussed by Bethe & Morrison (1956) and Evans (1955), to determine $\langle \sigma v \rangle$ as a function of $E_6^{1/2}$ and have found from a least-squares analysis that $\dot{\mathcal{S}}(0)/\mathcal{S}(0) = -2.60\ \text{MeV}^{-1/2}$ and $\frac{1}{2}\ddot{\mathcal{S}}(0)/\mathcal{S}(0) = 3.32\ \text{MeV}^{-1}$ as given in the Table. Thus Equation 33 for this case becomes

$$\sigma v = 7.30 \times 10^{-20}\,(1 - 2.60\, E_6^{1/2} + 3.32\, E_6)\ \text{cm}^3\ \text{sec}^{-1} \qquad 36.$$

With the coefficients of $T_9^{1/2}$, T_9, and $\langle 01 \rangle$ listed in Table II, Equation 34 becomes

$$\langle pn \rangle_\gamma = 7.30 \times 10^{-20}\,(1 - 0.860\, T_9^{1/2} + 0.429\, T_9)\ \text{cm}^3\ \text{sec}^{-1} \qquad 37.$$

while with the coefficient of [01] listed in Table II one finds

$$[pn]_\gamma = 4.40 \times 10^4\, \rho\,(1 - 0.860\, T_9^{1/2} + 0.429\, T_9)\ \text{sec}^{-1} \qquad 38.$$

Note that we append the lighter of the reaction products, in this case a photon, as a subscript in Equations 37 and 38 and in many equations to follow.

Equation 16 and the appropriate entries in the Table can now be employed to determine the photodisintegration rate for deuterium which becomes

$$\frac{1}{\tau_\gamma(\mathrm{D}^2)} = \lambda_\gamma(\mathrm{D}^2) = 2.07 \times 10^{14}\, T_9^{3/2}\,(1 - 0.860\, T_9^{1/2} + 0.429\, T_9) \times \exp(-25.82/T_9)\ \mathrm{sec}^{-1} \qquad 39.$$

Similar use of Equations 4 and 28 and appropriate entries in Table II yields

$$\tau_{n\gamma}(\mathrm{H}^1) = \frac{2.29 \times 10^{-5}}{\rho X_n}(1 - 0.860\, T_9^{1/2} + 0.429\, T_9)^{-1}\ \mathrm{sec} = \frac{1.37 \times 10^{19}}{n_n}(1 - 0.860\, T_9^{1/2} + 0.429\, T_9)^{-1}\ \mathrm{sec} \qquad 40.$$

In the second form of Equation 40 the number density of neutrons rather than the abundance by mass appears since for neutrons this is usually the convenient quantity. For the neutron lifetime one has

$$\tau_{p\gamma}(n) = \frac{2.29 \times 10^{-5}}{\rho X_\mathrm{H}}(1 - 0.860\, T_9^{1/2} + 0.429\, T_9)^{-1}\ \mathrm{sec} \qquad 41.$$

and for the energy generation rate

$$\mathcal{E}(pn)_\gamma = 9.29 \times 10^{22}\, \rho X_\mathrm{H} X_n\,(1 - 0.860\, T_9^{1/2} + 0.429\, T_9)\ \mathrm{erg\ g^{-1}\ sec^{-1}} \qquad 42.$$

For the reverse of a reaction such as $\mathrm{He}^3(n,p)\mathrm{T}^3$, for which there are two nuclei as reaction products, Equation 39 is replaced by

$$[p\mathrm{T}^3]_n = 7.07 \times 10^8\, \rho(1 - 0.597\, T_9^{1/2} + 0.183\, T_9)\exp(-8.864/T_9)\ \mathrm{sec}^{-1} \qquad 43.$$

and it is also possible to write

$$\langle p\mathrm{T}^3\rangle_n = 1.17 \times 10^{-15}\,(1 - 0.597\, T_9^{1/2} + 0.183\, T_9)\exp(-8.864/T_9)\ \mathrm{cm^3\ sec^{-1}} \qquad 44.$$

Note that the density ρ occurs in Equations 38 and 43 but not in Equations 37, 39, and 44. In equations for [234] and $\mathcal{E}_{234}$, ρ^2 appears. Equations 36 through 44 are the typical nonresonant neutron relations referred to in Table II. With this "do-it-yourself kit" the reader can construct appropriate equations for other reactions, proceeding in either direction, using the data given in Table II. It will be noted that in many cases either $\dot{S}$ or $\ddot{S}$ or both are zero, in which case the terms in $T_9^{1/2}$ or T_9 or both will not occur in Equations 36 through 43.

In Table II and subsequent tables we use the symbol $[\lambda_\gamma]$ which is to be read as [23], [234], or λ_γ, whichever is appropriate to the reaction under consideration. All of these quantities have the dimensions sec^{-1}. In the case of two nuclear reaction products the tables contain coefficients for both $\langle 23\rangle$ and [23]. In the case of reactions producing photons, the tables contain coefficients for only λ_γ. In the case of reactions with three final products the tables contain coefficients for only [234]. The quantity $\langle 234\rangle$ in units $\mathrm{cm^6\ sec^{-1}}$ can be calculated from $\langle 234\rangle = [234]/\rho^2 N_A{}^2$.

We emphasize that the tables give only the coefficients and that the complete formulae must be constructed in the manner in which we have just obtained Equations 37 through 44. In reactions involving identical particles it is most important to include the factor $(1+\delta_{01})^{-1}$ *in calculating the reaction rate* P_{01}

from ⟨01⟩ using Equation 1 or from [01] using Equation 7. Similar remarks apply in calculating P_{23} *using Equation 14 and* P_{234} *using Equation 23.*

The last entries in the tables require some comment. Our estimates for the uncertainty (standard deviation *not* limit of error) in the reaction rates and lifetimes have been chosen rather pessimistically and apply for the full range of temperature listed immediately below the uncertainty. The abbreviation FAC 2 is to be read as *factor of 2*. It will be clear from the experimental standard deviations given for the σ_{th} that in many cases the uncertainty is considerably smaller over a limited range which might typically cover $0 < T_9 \lesssim 0.1$. Detailed study of the available experimental data is required if it is desired to use uncertainties substantially smaller than the ones we have listed.

The upper limit cited in Table II for the temperature range over which the rates and lifetimes are valid is in most cases determined by lack of knowledge of the cross section above a maximum energy E_{max}, up to which observations have been made, or by failure of the expansion in Equation 33 to hold above this energy. The resonant energy of the first excited state in the compound nucleus above threshold, whose properties are unknown, sets a limiting E_{max}. In any case we take $T_{max} \sim E_{max}/k$; a more conservative choice would be a fraction, say one half, of this value. In ideal cases the background nonresonant cross section can be isolated from resonance effects up to large E_{max} and thus large T_{max}. In this case the data of Table II must be supplemented by those in Table IV which will be discussed below. In fact, it is in general necessary to employ both Table II and Table IV to obtain full information on the rate of a given reaction.

The references listed in the Table are designated by a notation in common use, e.g. Ja 57, and can be identified in detail by recourse to the list of literature cited at the end of the article. In many cases we have made a systematic least-squares analysis of available data so that our tabulated results may not agree with the results for $\mathcal{S}(0)$, for example, given by the authors quoted. In other cases we have made the first analysis of experimental data in which case we list this present paper as Fo 67. In order to keep the number of references to a minimum, we have in general cited nuclear-data tabulations where critical references to the original literature are given. In a few cases we cite references on proton-induced mirror reactions. Thus, for example, data on $D^2(p,\gamma)He^3$ given by Griffiths, Lal & Scarfe (1963) have been analyzed by us in such a way that the Coulomb-barrier factor could be removed to yield the energy dependence of the rate of the $D^2(n,\gamma)T^3$ reaction.

Nonresonant cross sections, charged particles.—At low energy, charged-particle interactions are dominated by Coulomb-barrier penetration factors. This has been well known since the pioneer work of Gamow (1928) and of Gurney & Condon (1928, 1929) many years ago. The low-energy relation for neutrons, $\sigma = \mathcal{S}(v)/v$, is replaced by

$$\sigma = \frac{S(E)}{E} \exp - (E_G/E)^{1/2} \qquad 45.$$

where

$$E_G = (2\pi\alpha Z_0 Z_1)^2 (Mc^2/2) \qquad 46.$$

is the "Gamow energy." Numerically one finds

$$\begin{aligned} E_G^{1/2} &= 0.98948\, Z_0 Z_1 A_1^{1/2}\ \mathrm{MeV}^{1/2}\ \mathrm{LAB} \\ &= 0.98948\, Z_0 Z_1 A^{1/2}\ \mathrm{MeV}^{1/2}\ \mathrm{CM} \end{aligned} \qquad 47.$$

It is most important to note that charged-particle cross sections vary in the same way at low energy for all partial waves, in marked contrast to neutron cross sections where $\Gamma_n(l) \propto (kR)^{2l+1} \propto (E/E_R)^{l+1/2}$ where $k = Mv/\hbar$ is the wavenumber, R the radius of interaction, and E_R the centrifugal barrier energy $= \hbar^2/2MR^2$. In charged-particle widths the l dependence is separable as an approximately energy-independent factor which decreases fairly rapidly with increasing l. In addition the v dependence of the partial width is canceled by a term in v^{-1} in the Coulomb-barrier penetration factor, leaving the E^{-1} term from $\pi\lambda\!\!\!^{-2}$ and the Gamow exponential in first approximation. In Equation 45 the *cross-section factor* $S(E)$ includes the energy dependence of the penetration factor not included in the Gamow exponential plus that of the intrinsic nuclear factors governing the cross section. Far from a nuclear resonance, $S(E)$ is a slowly varying function of E and can be conveniently expressed as the first three terms of a Maclaurin series in the center-of-momentum energy E. Thus

$$S(E) = S(0)\left(1 + \frac{S'(0)}{S(0)}E + \frac{1}{2}\frac{S''(0)}{S(0)}E^2\right) \qquad 48.$$

where the prime indicates differentiation with respect to E. If E is in MeV and σ is in barns in Equation 45, then $S(0)$ is in MeV-barn, $S'(0)$ is in barns, and $S''(0)$ in barn-MeV^{-1}. In this article all tabulated values for S refer to center-of-momentum coordinates.

Substitution of Equations 45 and 48 into Equation 32 yields (Caughlan & Fowler 1962, Bahcall 1966)

$$\begin{aligned} \langle\sigma v\rangle &= \frac{(8/\pi)^{1/2}}{M^{1/2}(kT)^{3/2}}\int S(E)\exp(-E_G^{1/2}/E^{1/2} - E/kT)dE \\ &= \left(\frac{2}{M}\right)^{1/2}\frac{\Delta E_0}{(kT)^{3/2}}S_{\mathrm{eff}}\exp(-\tau) \end{aligned} \qquad 49.$$

so that

$$\langle 01\rangle = \{1.3006 \times 10^{-14}(Z_0Z_1/A)^{1/3}S_{\mathrm{eff}}\}\, T_9^{-2/3}\exp(-\tau)\ \mathrm{cm}^3\ \mathrm{sec}^{-1} \qquad 50.$$

and

$$[01] = \{7.8327 \times 10^9 (Z_0Z_1/A)^{1/3}S_{\mathrm{eff}}\}\,\rho\, T_9^{-2/3}\exp(-\tau)\ \mathrm{sec}^{-1} \qquad 51.$$

where

$$S_{\text{eff}} = S(0)\left[1 + \frac{5}{12\tau} + \frac{S'(0)}{S(0)}\left(E_0 + \frac{35}{36}kT\right) + \frac{1}{2}\frac{S''(0)}{S(0)}\left(E_0^2 + \frac{89}{36}E_0kT\right)\right] \text{ MeV-barn} \quad 52.$$

$$= S(0)\left[1 + 9.807 \times 10^{-2} W^{-1/3} T_9^{1/3} + 0.1220 \frac{S'(0)}{S(0)} W^{1/3} T_9^{2/3} + 8.378 \times 10^{-2} \frac{S'(0)}{S(0)} T_9 + 7.447 \times 10^{-3} \frac{S''(0)}{S(0)} W^{2/3} T_9^{4/3} + 1.300 \times 10^{-2} \frac{S''(0)}{S(0)} W^{1/3} T_9^{5/3}\right] \quad 53.$$

$$W = Z_0^2 Z_1^2 A \quad 54.$$

$$\tau = 3E_0/kT = 3[\pi\alpha Z_0 Z_1 (Mc^2/2kT)^{1/2}]^{2/3} = 4.2487\, W^{1/3}\, T_9^{-1/3} \quad 55.$$

$$E_0 = [\pi\alpha Z_0 Z_1 kT\, (Mc^2/2)^{1/2}]^{2/3} = 0.12204\, W^{1/3}\, T_9^{2/3} \text{ MeV} \quad 56.$$

$$kT = 0.08617\, T_9 = T_9/11.605 \text{ MeV} \quad 57.$$

$$\Delta E_0 = 4(E_0 kT/3)^{1/2} = \frac{2^{11/6}}{3^{1/2}} (\pi\alpha Z_0 Z_1)^{1/3} (Mc^2)^{1/6} (kT)^{5/6} = 0.23682\, W^{1/6}\, T_9^{5/6} \text{ MeV} \quad 58.$$

$$\alpha = e^2/\hbar c = (137.0388)^{-1} \quad 59.$$

In the above equations E_0 is the effective interaction energy given by the energy at which the maximum occurs in the product of the Gamow and Maxwell-Boltzmann exponentials in the integrand in the first form of Equation 49. It is frequently convenient to know the temperature corresponding to a given E_0 in MeV. This temperature is given by

$$T_0 = 23.46\, W^{-1/2}\, (E_6)_0^{3/2} \quad 60.$$

The quantity ΔE_0 is the effective energy interval given by the full width between the points at $1/e$ times the maximum of the Gamow-Maxwell-Boltzmann product. Reference should be made to Fowler & Vogl (1964) for further details. Data for the nonresonant portion of certain charged-particle reactions are given in Table III. The numerical coefficients listed for ⟨01⟩ and [01] in Table III are evaluated by use of the factors in curly brackets in Equations 50 and 51 respectively with S_{eff} replaced by $S(0)$. The data in Table III must frequently be supplemented by the resonant and continuum data given respectively in Tables V and VI discussed below. Illustrative examples of the use of Tables III, V, and VI will be discussed after all necessary preliminaries have been given.

Resonant cross sections, neutrons, and charged particles.—A single resonance in the cross section of a nuclear reaction $(0+1 \rightarrow 2+3+Q)$ can be represented most simply as a function of energy in terms of the classical, Breit-Wigner formula (see p. 544):

TABLE III

Charged-Particle Nonresonant-Reaction Data

Reaction		$H^1(p,e^+\nu)D^2$	$H^1(pe^-,\nu)D^2$	$He^3(p,e^+\nu)He^4$	$D^2(p,\gamma)He^3$	$D^2(d,n)He^3$	$D^2(d,p)T^3$
Parameters in typical equations indicated in parentheses							
Q	(8)	1.442[a]	1.442[b]	19.796[c]	5.494	3.269	4.033
$(Q/k)_9$	(80)(83)	1.674E 01	1.674E 01	2.297E 02	6.375E 01	3.794E 01	4.680E 01
$E_G^{1/2}$ LAB	(47)	9.931E−01	9.931E−01	1.986	9.931E−01	1.404	1.404
$E_G^{1/2}$ CM	(47)	7.022E−01	7.022E−01	1.720	8.108E−01	9.928E−01	9.928E−01
$E_0/T_9^{2/3}$	(56)	9.709E−02	9.709E−02	1.764E−01	1.069E−01	1.223E−01	1.223E−01
$T_0/E_0^{3/2}$	(60)	3.305E 01	3.305E 01	1.350E 01	2.863E 01	2.338E 01	2.338E 01
$\Delta E_0/T_9^{5/6}$	(58)	2.112E−01	2.112E−01	2.847E−01	2.216E−01	2.371E−01	2.371E−01
$\tau T_9^{1/3}$	(55)(78)	3.380	3.380	6.141	3.720	4.258	4.258
$S(0)$	(53)	3.36 E−25	1.15 E−30[d]	3.70 E−23	2.50 E−07	5.30 E−02	5.30 E−02
$S'(0)/S(0)$	(53)	8.04	—	—	3.16 E 01	4.95	4.95
Coefficients in typical equations indicated in parentheses							
$T_9^{1/3}$	(78)	1.23 E−01	—	6.78 E−02	1.12 E−01	9.79 E−02	9.79 E−02
$T_9^{2/3}$	(78)	7.80 E−01	—	—	3.38	6.06 E−01	6.06 E−01
T_9	(78)	6.73 E−01	—	—	2.65	4.15 E−01	4.15 E−01
⟨01⟩	(82)	5.49 E−39	1.88 E−44[d]	6.66 E−37	3.71 E−21	6.88 E−16	6.88 E−16
[01]	(78)	3.31 E−15	1.13 E−20[d]	4.01 E−13	2.24 E 03	4.14 E 08	4.14 E 08
⟨23⟩	(83)	—	—	—	—	1.19 E−15	1.19 E−15
[λ_γ]	(80)(83)		—	—	3.65 E 13	7.17 E 08	7.19 E 08
$\tau_1(0)$	(79)	3.05 E 14	8.91 E 19[e]	2.51 E 12	4.51 E−04	4.86 E−09	4.86 E−09
$\tau_0(1)$	(79)	3.05 E 14	8.91 E 19[e]	7.52 E 12	9.01 E−04	4.86 E−09	4.86 E−09
$\mathcal{E}_{01}$	(81)	2.27[a]E 03	7.75[b]E−03[d]	2.52[c]E 06	5.84 E 21	1.61 E 26	1.99 E 26
Uncertainty		±10%	±50%	FAC 3	±10%	±10%	±10%
T_9 limits		0 to 5	0 to 5	0 to 5	0 to 5	0 to 10	0 to 10
References		Pa 64	Ba 64	We 67	Pa 64	Pa 64	Pa 64

Reaction		$D^2(d,\gamma)He^4$	$T^3(p,\gamma)He^4$	$T^3(d,n)He^4$	$He^3(d,p)He^4$	$He^4(t,\gamma)Li^7$	$He^4(\tau,\gamma)Be^7$
Parameters in typical equations indicated in parentheses							
Q	(8)	23.847	19.814	17.590	18.354	2.467	1.587
$(Q/k)_9$	(80)(83)	2.767E 02	2.299E 02	2.041E 02	2.130E 02	2.863E 01	1.842E 01
$E_G^{1/2}$ LAB	(47)	1.404	9.931E−01	1.404	2.808	3.437	6.872
$E_G^{1/2}$ CM	(47)	9.928E−01	8.598E−01	1.087	2.174	2.595	5.190
$E_0/T_9^{2/3}$	(56)	1.223E−01	1.111E−01	1.299E−01	2.063E−01	2.321E−01	3.684E−01
$T_0/E_0^{3/2}$	(60)	2.338E 01	2.699E 01	2.135E 01	1.067E 01	8.944	4.472
$\Delta E_0/T_9^{5/6}$	(58)	2.371E−01	2.260E−01	2.444E−01	3.079E−01	3.266E−01	4.115E−01
$\tau T_9^{1/3}$	(55)(78)	4.258	3.869	4.524	7.181	8.080	1.283E 01
$S(0)$	(53)	2.22 E−10	2.56 E−06	1.10 E 01	7.00	6.40 E−05	4.70 E−04
$S'(0)/S(0)$	(53)	1.19 E 02	1.51 E 01	1.38 E 01	−4.97	—	−5.96 E−01
$\frac{1}{2}S''(0)/S(0)$	(53)	1.37 E 02	4.46 E 01	6.23 E 02	7.10 E 01	—	1.97 E−01
Coefficients in typical equations indicated in parentheses							
$T_9^{1/3}$	(78)	9.79 E−02	1.08 E−01	9.21 E−02	5.80 E−02	5.16 E−02	3.25 E−02
$T_9^{2/3}$	(78)	1.45 E 01	1.68	1.80	−1.03	—	−2.19 E−01
T_9	(78)	9.92	1.26	1.16	−4.16 E−01	—	−4.99 E−02
$T_9^{4/3}$	(78)	2.04	5.51 E−01	1.05 E 01	3.02	—	2.67 E−02
$T_9^{5/3}$	(78)	3.56	1.06	1.72 E 01	3.12	—	1.54 E−02
⟨01⟩	(82)	2.88 E−24	3.65 E−20	1.34 E−13	1.08 E−13	8.75 E−19	8.10 E−18
[01]	(78)	1.73	2.20 E 04	8.09 E 10	6.49 E 10	5.27 E 05	4.88 E 06
⟨23⟩	(83)	—	—	7.44 E−13	5.97 E−13	—	—
[λ_γ]	(80)(83)	7.86 E 10	5.74 E 14	4.48 E 11	3.60 E 11	5.87 E 15	5.43 E 16
$\tau_1(0)$	(79)	1.16	4.58 E−05	2.49 E−11	3.10 E−11	5.72 E−06	6.18 E−07
$\tau_0(1)$	(79)	1.16	1.37 E−04	3.73 E−11	4.65 E−11	7.59 E−06	8.21 E−07
$\mathcal{E}_{01}$	(81)	4.92 E 18	1.38 E 23	2.26 E 29	1.89 E 29	1.04 E 23	6.19 E 23
Uncertainty		FAC 2	±20%	0 to 20%	−10 to +70%	±20%	±20%
T_9 limits		0 to 10	0 to 10	0 to 0.15	0 to 0.65	0 to 10	0 to 10
References		Zu 63	Ja 57	Ja 57	Ja 57	Gr 61	Pa 64

Units: Energy in MeV; $S(0)$ in MeV-barn, $S'(0)/S(0)$ in MeV^{-1}, $\frac{1}{2}S''(0)/S(0)$ in MeV^{-2}; ⟨ ⟩ in cm^3 sec^{-1};] in sec^{-1}; τ in sec; $\mathcal{E}_{01}$ in erg g^{-1} sec^{-1}; $T_9 = T/10^9$.

[a] Neutrino loss ≈ 0.26 MeV. [b] Neutrino loss ≈ 1.44 MeV. [c] Neutrino loss ≈ 9.64 MeV.

(*Nota bene:* tabulated values for Q and $\mathcal{E}_{01}$ include neutrino energy.)

[d] Include an additional factor $\rho(1+X_H)/2T_9^{1/2}$. [e] Include an additional factor $2T_9^{1/2}/\rho(1+X_H)$.

TABLE III (*continued*)

CHARGED-PARTICLE NONRESONANT-REACTION DATA

Reaction		$T^3(t,2n)He^4$	$He^3(t,np)He^4$	$He^3(\tau,2p)He^4$	$He^3(t,d)He^4$	$Li^6(p,\tau)He^4$	$Li^7(p,\alpha)He^4$
		Parameters in typical equations indicated in parentheses					
Q	(8)	11.332	12.096	12.860	14.321	4.021	17.347
$(Q/k)_9$	(80)(83)	1.315E 02	1.404E 02	1.492E 02	1.662E 02	4.667E 01	2.013E 02
$E_G^{1/2}$ LAB	(47)	1.718	3.437	6.872	3.437	2.979	2.979
$E_G^{1/2}$ CM	(47)	1.215	2.430	4.860	2.430	2.757	2.786
$E_0/T_9^{2/3}$	(56)	1.399E−01	2.221E−01	3.526E−01	2.221E−01	2.417E−01	2.433E−01
$T_0/E_0^{3/2}$	(60)	1.910E 01	9.552	4.776	9.552	8.418	8.331
$\Delta E_0/T_9^{5/6}$	(58)	2.536E−01	3.195E−01	4.025E−01	3.195E−01	3.333E−01	3.344E−01
$\tau T_9^{1/3}$	(55)(78)	4.872	7.733	1.228E 01	7.733	8.413	8.471
$S(0)$	(53)	1.60 E−01	6.50 E−01	5.00	4.50 E−01	5.50	1.25 E−01
		Coefficients in typical equations indicated in parentheses					
$T_9^{1/3}$	(78)	8.55 E−02	5.39 E−02	3.39 E−02	5.39 E−02	4.95 E−02	4.92 E−02
⟨01⟩	(82)	1.81 E−15	9.29 E−15	9.00 E−14	6.43 E−15	1.08 E−13	2.45 E−15
[01]	(78)	1.09 E 09	5.59 E 09	5.42 E 10	3.87 E 09	6.53 E 10	1.47 E 09
⟨23⟩	(83)	—	—	—	1.03 E−14	1.16 E−13	1.15 E−14
$[\lambda_\gamma]$	(84)(83)	3.70 E−01	1.90	1.84 E 01	6.19 E 09	6.97 E 10	6.91 E 09
$\tau_1(0)$	(79)	2.76 E−09	5.39 E−10	5.56 E−11	7.79 E−10	1.54 E−11	6.84 E−10
$\tau_0(1)$	(79)	2.76 E−09	5.39 E−10	5.56 E−11	7.79 E−10	9.22 E−11	4.76 E−09
$\mathcal{E}_{01}$	(81)	6.57 E 26	7.18 E 27	3.70 E 28	5.88 E 27	4.18 E 28	3.49 E 27
Uncertainty		±30%	±30%	±40%	±30%	±10%	±10%
T_9 limits		0 to 10	0 to 10	0 to 10	0 to 10	0 to 5	0 to 5
References		Go 62	Yo 61	Wi 67	Yo 61	Ja 57	Ja 57

Reaction		$C^{12}(p,\gamma)N^{13}$	$C^{13}(p,\gamma)N^{14}$	$N^{13}(p,\gamma)O^{14}$	$N^{14}(p,\gamma)O^{15}$	$N^{15}(p,\gamma)O^{16}$	$N^{15}(p,\alpha)C^{12}$
		Parameters in typical equations indicated in parentheses					
Q	(8)	1.944	7.550	4.626	7.293	12.126	4.965
$(Q/k)_9$	(80)(83)	2.256E 01	8.762E 01	5.369E 01	8.463E 01	1.407E 02	5.761E 01
$E_G^{1/2}$ LAB	(47)	5.958	5.958	6.952	6.952	6.952	6.952
$E_G^{1/2}$ CM	(47)	5.723	5.740	6.697	6.714	6.729	6.729
$E_0/T_9^{2/3}$	(56)	3.932E−01	3.940E−01	4.367E−01	4.374E−01	4.381E−01	4.381E−01
$T_0/E_0^{3/2}$	(60)	4.056	4.043	3.466	3.457	3.449	3.449
$\Delta E_0/T_9^{5/6}$	(58)	4.251E−01	4.255E−01	4.480E−01	4.484E−01	4.487E−01	4.487E−01
$\tau T_9^{1/3}$	(55)(78)	1.369E 01	1.372E 01	1.520E 01	1.523E 01	1.525E 01	1.525E 01
$S(0)$	(53)	1.40 E−03	5.50 E−03	2.75 E−03[a]	2.75 E−03	2.74 E−02	5.34 E 01
$S'(0)/S(0)$	(53)	3.04	2.43	—	—	6.79	1.54 E 01
$\frac{1}{2}S''(0)/S(0)$	(53)	1.33 E 01	8.97	—	—	—	—
		Coefficients in typical equations indicated in parentheses					
$T_9^{1/3}$	(78)	3.04 E−02	3.04 E−02	2.74 E−02	2.74 E−02	2.73 E−02	2.73 E−02
$T_9^{2/3}$	(78)	1.19	9.58 E−01	—	—	2.97	6.74
T_9	(78)	2.54 E−01	2.04 E−01	—	—	5.69 E−01	1.29
$T_9^{4/3}$	(78)	2.06	1.39	—	—	—	—
$T_9^{5/3}$	(78)	1.12	7.53 E−01	—	—	—	—
⟨01⟩	(82)	3.39 E−17	1.33 E−16	7.00 E−17	6.99 E−17	6.95 E−16	1.35 E−12
[01]	(78)	2.04 E 07	8.01 E 07	4.21 E 07	4.21 E 07	4.19 E 08	8.16 E 11
⟨23⟩	(83)	—	—	—	—	—	9.56 E−13
$[\lambda_\gamma]$	(80)(83)	1.81 E 17	9.53 E 17	1.50 E 18	1.14 E 18	1.52 E 19	5.76 E 11
$\tau_1(0)$	(79)	4.93 E−08	1.26 E−08	2.39 E−08	2.40 E−08	2.41 E−09	1.24 E−12
$\tau_0(1)$	(79)	5.87 E−07	1.62 E−07	3.09 E−07	3.33 E−07	3.58 E−08	1.84 E−11
$\mathcal{E}_{01}$	(81)	3.17 E 24	4.45 E 25	1.44 E 25	2.10 E 25	3.24 E 26	2.58 E 29
Uncertainty		0 to −20%	0 to −20%	FAC 10	±10%	±20%	±20%
T_9 limits		0 to 0.55	0 to 0.65	0 to 1.5	0 to 2	0 to 1	0 to 1
References		Vo 63	Vo 63	Fo 67	He 63	He 60, Ca 62	He 60, Ca 62

Units: Energy in MeV; $S(0)$ in MeV-barn, $S'(0)/S(0)$ in MeV^{-1}, $\frac{1}{2}S''(0)/S(0)$ in MeV^{-2}; ⟨ ⟩ in cm^3 sec^{-1}; [ ] in sec^{-1}; τ in sec; $\mathcal{E}_{01}$ in erg g^{-1} sec^{-1}; $T_9 = T/10^9$.

[a] Assumed equal to $S(0)$ for $N^{14}(p,\gamma)O^{15}$.

TABLE III (*concluded*)

CHARGED-PARTICLE NONRESONANT-REACTION DATA

Reaction		$Be^7(p,\gamma)B^8$	$Be^9(p,d)Be^8$	$Be^9(p,\alpha)Li^6$	$B^{11}(p,2\alpha)He^4$	$N^{14}(\alpha,\gamma)F^{18}$	$O^{18}(\alpha,\gamma)Ne^{22}$
		Parameters in typical equations indicated in parentheses					
Q	(8)	0.135	0.559	2.126	8.682	4.416	9.667
$(Q/k)_9$	(80)(83)	1.564	6.492	2.468E 01	1.008E 02	5.125E 01	1.122E 02
$E_G^{1/2}$ LAB	(47)	3.972	3.972	3.972	4.965	2.771E 01	3.167E 01
$E_G^{1/2}$ CM	(47)	3.715	3.767	3.767	4.753	2.444E 01	2.864E 01
$E_0/T_9^{2/3}$	(56)	2.948E−01	2.976E−01	2.976E−01	3.474E−01	1.035	1.151
$T_0/E_0^{3/2}$	(60)	6.248	6.161	6.161	4.884	9.498E−01	8.103E−01
$\Delta E_0/T_9^{5/6}$	(58)	3.681E−01	3.698E−01	3.698E−01	3.996E−01	6.897E−01	7.272E−01
$\tau T_9^{1/3}$	(55)(78)	1.026E 01	1.036E 01	1.036E 01	1.209E 01	3.603E 01	4.006E 01
$S(0)$	(53)	4.00 E−05	1.50 E 01	1.50 E 01	1.00 E 02	8.73 E 06	7.77 E 07
		Coefficients in typical equations indicated in parentheses					
$T_9^{1/3}$	(78)	4.06 E−02	4.02 E−02	4.02 E−02	3.45 E−02	1.16 E−02	1.04 E−02
⟨01⟩	(82)	8.61 E−19	3.20 E−13	3.20 E−13	2.28 E−12	1.87 E−07	1.71 E−06
[01]	(78)	5.19 E 05	1.93 E 11	1.93 E 11	1.38 E 12	1.13 E 17	1.03 E 18
⟨23⟩	(83)	—	3.61 E−13	1.98 E−13	—	—	—
$[\lambda_\gamma]$	(80)(83)	6.77 E 15	2.17 E 11	1.19 E 11	4.82 E 02	6.11 E 27	6.04 E 28
$\tau_1(0)$	(79)	1.94 E−06	5.23 E−12	5.23 E−12	7.33 E−13	3.55 E−17	3.88 E−18
$\tau_0(1)$	(79)	1.35 E−05	4.68 E−11	4.68 E−11	8.00 E−12	1.24 E−16	1.74 E−17
$\mathcal{E}_{01}$	(81)	9.54 E 21	1.15 E 28	4.35 E 28	1.04 E 30	8.58 E 33	1.34 E 35
Uncertainty		±20%	±30%	±30%	±10%	FAC 3	FAC 3
T_9 limits		0 to 10	0 to 2	0 to 2	0 to 3	0 to 2	0 to 2
References		Pa 66	Mi 54	Mi 54	Sc 51	Ca 64	Ca 64

Reaction		$B^{10}(p,\alpha)Be^7$	$C^{13}(\alpha,n)O^{16}$	$O^{16}(p,\gamma)F^{17}$	$O^{16}(\alpha,\gamma)Ne^{20}$	$O^{17}(p,\alpha)N^{14}$	$Ne^{20}(p,\gamma)Na^{21}$
		Parameters in typical equations indicated in parentheses					
Q	(8)	1.148	2.214	0.601	4.730	1.193	2.432
$(Q/k)_9$	(80)(83)	1.332E 01	2.570E 01	6.969	5.489E 01	1.384E 01	2.823E 01
$E_G^{1/2}$ LAB	(47)	4.965	2.375E 01	7.945	3.167E 01	7.945	9.931
$E_G^{1/2}$ CM	(47)	4.733	2.077E 01	7.706	2.832E 01	7.719	9.690
$E_0/T_9^{2/3}$	(56)	3.464E−01	9.286E−01	4.795E−01	1.142	4.800E−01	5.586E−01
$T_0/E_0^{3/2}$	(60)	4.904	1.118	3.012	8.195E−01	3.007	2.395
$\Delta E_0/T_9^{5/6}$	(58)	3.990E−01	6.533E−01	4.694E−01	7.244E−01	4.697E−01	5.067E−01
$\tau T_9^{1/3}$	(55)(78)	1.206E 01	3.233E 01	1.669E 01	3.976E 01	1.671E 01	1.945E 01
$S(0)$	(53)	9.12	5.48 E 05	1.03 E−02	1.00 E−01	1.20 E−01	5.50 E−02
$S'(0)/S(0)$	(53)	−1.44	2.20	−2.73	—	—	7.64 E−01
$\frac{1}{2}S''(0)/S(0)$	(53)	2.50	—	2.93	—	—	—
		Coefficients in typical equations indicated in parentheses					
$T_9^{1/3}$	(78)	3.45 E−02	1.29 E−02	2.50 E−02	1.05 E−02	−3.68[a]	2.14 E−02
$T_9^{2/3}$	(78)	−4.98 E−01	2.04	−1.31	—	—	4.27 E−01
T_9	(78)	−1.21 E−01	1.84 E−01	−2.29 E−01	—	—	6.40 E−02
$T_9^{4/3}$	(78)	3.00 E−01	—	6.73 E−01	—	—	—
$T_9^{5/3}$	(78)	1.85 E−01	—	2.99 E−01	—	—	—
⟨01⟩	(82)	2.09 E−13	1.12 E−08	2.73 E−16	2.22 E−15	3.17 E−15	1.56 E−15
[01]	(78)	1.26 E 11	6.77 E 15	1.64 E 08	1.34 E 09	1.91 E 09	9.41 E 08
⟨23⟩	(83)	1.57 E−13	6.51 E−08	—	—	2.14 E−15	—
$[\lambda_\gamma]$	(80)(83)	9.48 E 10	3.92 E 16	4.98 E 17	7.57 E 19	1.29 E 09	4.36 E 18
$\tau_1(0)$	(79)	8.01 E−12	5.91 E−16	6.14 E−09	2.99 E−09	5.27 E−10	1.07 E−09
$\tau_0(1)$	(79)	7.96 E−11	1.92 E−15	9.74 E−08	1.19 E−08	8.89 E−09	2.12 E−08
$\mathcal{E}_{01}$	(81)	1.38 E 28	2.78 E 32	5.90 E 24	9.55 E 25	1.28 E 26	1.10 E 26
Uncertainty		±20%	±30%	±30%	FAC 3	FAC 2	±50%
T_9 limits		0 to 5	0 to 1	0 to 2	0 to 0.2	0.006 to 0.02	0 to 2
References		Ja 57	Da 66	Ch Du 61	Fo 64	Br 62b	Ta 59

Units: Energy in MeV; $S(0)$ in MeV-barn, $S'(0)/S(0)$ in MeV^{-1}, $\frac{1}{2}S''(0)/S(0)$ in MeV^{-2}; ⟨ ⟩ in cm^3 sec^{-1}; [ ] in sec^{-1}; τ in sec; $\mathcal{E}_{01}$ in erg g^{-1} sec^{-1}; $T_9 = T/10^9$.

[a] Best empirical adjustment.

$$\sigma = \frac{\pi\hbar^2}{2ME}\frac{\omega_r\Gamma_1\Gamma_2}{(E-E_r)^2+\Gamma^2/4}$$
$$= \frac{0.6566}{AE}\frac{\omega_r\Gamma_1\Gamma_2}{(E-E_r)^2+\Gamma^2/4}\text{ barn } (E \text{ in MeV}) \qquad 61.$$

where E_r is the resonance energy in the center-of-momentum system for particles $0+1$, Γ_1 is the partial width for the decay of the resonant state by re-emission of $0+1$, Γ_2 is the partial width for emission of $2+3$, $\Gamma=\Gamma_1+\Gamma_2+\ldots$ is the sum over all partial widths, and $\omega_r=(1+\delta_{01})g_r/g_0g_1$ with $g_r=2J_r+1$, J_r being the spin of the resonant state.

In Tables IV and V a given resonance is most directly identified in the literature in terms of the resonance energy in the laboratory system (E_r LAB), but it must be remembered that the resonance energy in the center-of-momentum system (E_r CM) has been used in determining $(E_r/k)_9$ in the tables. A second means of identification is the excitation energy of the state in the compound nucleus of the reaction and this is listed as E_x in Tables IV and V.

It is customary to evaluate $\langle\sigma v\rangle$ for the cross section given by Equation 61 in the approximation that the resonance is sharp; that is, the full width at resonance, Γ_r, is considerably less than the effective spread in energy of the interacting particles. This effective spread in energy is kT when neutrons are involved, ΔE_0 for two charged particles. In this approximation it is a matter of elementary integration to show that

$$\langle\sigma v\rangle = \left(\frac{2\pi\hbar^2}{MkT}\right)^{3/2}\frac{(\omega\gamma)_r}{\hbar}\exp(-E_r/kT) \qquad 62.$$

so that

$$\langle 01\rangle = \{2.557\times 10^{-13}A^{-3/2}(\omega\gamma)_r\}T_9^{-3/2}\exp(-11.605\,E_r/T_9)\text{ cm}^3\text{ sec}^{-1} \qquad 63.$$

and

$$[01] = \{1.540\times 10^{11}\,A^{-3/2}(\omega\gamma)_r\}\rho T_9^{-3/2}\exp(-11.605\,E_r/T_9)\text{ sec}^{-1} \qquad 64.$$

where $(\omega\gamma)_r=\omega_r\gamma_r=(\omega\Gamma_1\Gamma_2/\Gamma)_r$.

The numerical coefficients listed for $\langle 01\rangle$ and [01] in Tables IV and V are evaluated by the use of the factors in curly brackets in Equations 63 and 64 respectively. A useful expression for $\lambda_\gamma(2)$ for resonant capture reactions, which can be derived for Equations 16 and 62, is

$$\lambda_\gamma(2) = \frac{g_r\gamma_r}{g_2\hbar}\exp\left(-\frac{Q+E_r}{kT}\right) = \frac{(\omega\gamma)_r}{\omega_2\hbar}\exp\left(-\frac{Q+E_r}{kT}\right)\text{ sec}^{-1} \qquad 65.$$

where $\omega_2=(1+\delta_{01})g_2/g_0g_1$.

In the above equations the quantity $(\omega\gamma)_r$ is evaluated at resonance and can be determined even in a "poor" resolution experiment where the uncertainty in energy is greater than Γ_r, by experimentally determining the integral of the cross section over energy and then using

$$(\omega\gamma)_r = (\pi^2\hbar^2/ME_r)^{-1}\int_r \sigma dE \qquad 66.$$

This expression can be derived easily by using Equation 61 and the definition of $(\omega\gamma)_r$ above. In "good" resolution experiments $(\omega\gamma)_r$ can be determined in terms of the energy E_r, the cross section σ_r, and the full width at half-maximum Γ_r, all as measured at resonance by the use of

$$\begin{aligned}(\omega\gamma)_r &= \frac{\sigma_r M\Gamma_r E_r}{2\pi\hbar^2} = 0.3807\ \sigma_r A(\Gamma_r E_r)_{\rm CM}\ \text{MeV} \\ &= 0.3807\left(\frac{A_0}{A_0+A_1}\right)^3 \sigma_r A_1(\Gamma_r E_r)_{\rm LAB}\ \text{MeV}\end{aligned} \qquad 67.$$

for σ_r in barns and Γ_r, E_r in MeV. In Tables IV and V, E_r, and $(\omega\gamma)_r$ are always listed and J_r^π, ω_r, σ_r, and Γ_r are listed when known. The superscript π designates the parity (+ for even, − for odd) of the resonant state. Along

TABLE IV

NEUTRON RESONANT-REACTION DATA

Reaction		$He^3(n,p)T^3$	$Li^6(n,t)He^4$	$Li^7(n,\gamma)Li^8$	$B^{11}(n,\gamma)B^{12}$	$N^{14}(n,p)C^{14}$	$N^{14}(n,p)C^{14}$
Parameters in typical equations indicated in parentheses							
Q	(8)	0.764	4.785	2.033	3.369	0.626	0.626
E_x		22.354	7.471	2.258	3.387	11.297	11.431
E_r LAB	(71)	2.370	0.255	0.258	0.020	0.495	0.639
E_r CM	(64)(67)	1.776	0.218	0.226	0.018	0.462	0.596
$(E_r/k)_9$	(72)	2.061E 01	2.534	2.618	0.213	5.358	6.917
E_r' CM	(68)	2.540	5.004	2.258	3.387	1.088	1.222
$(E_r'/k)_9$	(73)	2.947E 01	5.807E 01	2.621E 01	3.931E 01	1.263E 01	1.419E 01
J_r^π, l_r		2^-, 1	$5/2^-$, 1	3^+, 1	(3^+), (1)	$1/2^-$, 1	$1/2^+$, 0
ω_r	(64)	1.250	1.000	0.875	0.875	0.333	0.333
σ_r	(67)(70)	4.40 E−01	2.60	2.31 E−05	—	3.94 E−01	2.58 E−01
Γ_r	(67)	2.45	1.02 E−01	3.06 E−02	—	7.00 E−03	4.01 E−02
$(\omega\gamma)_r$	(64)(65)	5.51 E−01	1.90 E−02	5.36 E−08	3.66 E−07	4.56 E−04	2.21 E−03
v_r	(71)	2.13 E 09	6.98 E 08	7.03 E 08	1.96 E 08	9.73 E 08	1.11 E 09
$\mathcal{S}_r$	(70)	9.37 E−16	1.82 E−15	1.62 E−20	—	3.84 E−16	2.86 E−16
$\mathcal{S}(0)$	(69)	0	0	0	0	0	3.23 E−19
Coefficients in typical equations indicated in parentheses							
⟨01⟩	(72)	2.14 E−13	6.07 E−15	1.65 E−20	1.05 E−19	1.28 E−16	6.20 E−16
[01]	(72)	1.29 E 11	3.65 E 09	9.96 E 03	6.35 E 04	7.69 E 07	3.73 E 08
⟨23⟩	(73)	2.15 E−13	6.49 E−15	—	—	3.84 E−16	1.86 E−15
$[\lambda_\gamma]$	(73)	1.29 E 11	3.91 E 09	1.30 E 14	1.49 E 15	2.31 E 08	1.12 E 09
$\tau_1(0)$	(74)	7.81 E−12	2.76 E−10	1.01 E−04	1.59 E−05	1.31 E−08	2.70 E−09
$\tau_0(1)$	(74)	2.34 E−11	1.65 E−09	7.05 E−04	1.73 E−04	1.82 E−07	3.75 E−08
$\mathcal{E}_{01}$	(75)	3.13 E 28	2.78 E 27	2.76 E 21	1.86 E 22	3.29 E 24	1.60 E 25
Uncertainty		±10%	±10%	±50%	±50%	±50%	±50%
T_9 limits		3 to 10	0.4 to 10	1 to 5	0.01 to 3	0.5 to 5	0.5 to 5
References		Hu 58	Hu 60	Hu 60, La 66	Aj 67	Aj 59	Aj 59

Units: Energy in MeV; σ_r in barns; v_r in cm sec^{-1}; ($\mathcal{S}$) and ⟨ ⟩ in cm^3 sec^{-1}; [ ] in sec^{-1}; τ in sec; $\mathcal{E}_{01}$ in erg g^{-1} sec^{-1}. $T_9 = T/10^9$.

TABLE V

Charged-Particle Resonant-Reaction Data

Reaction		$T^3(d,n)He^4$	$He^3(d,p)He^4$	$Li^7(p,\gamma)Be^8$	$Li^7(\alpha,\gamma)B^{11}$	$Li^7(\alpha,\gamma)B^{11}$	$Li^7(\alpha,\gamma)B^{11}$
Parameters in typical equations indicated in parentheses							
Q	(8)	17.590	18.354	17.252	8.664	8.664	8.664
E_x		16.709	16.672	17.638	8.920	9.186	9.274
E_r LAB		0.128[a]	0.474[b]	0.441	0.401	0.819	0.958
E_r CM	(64)(77)	0.077	0.284	0.386	0.255	0.522	0.610
$(E_r/k)_9$	(78)	0.891	3.297	4.479	2.963	6.052	7.079
E_r' CM	(68)	17.666	18.638	17.638	8.920	9.186	9.274
$(E_r'/k)_9$	(80)(83)	2.050E 02	2.163E 02	2.047E 02	1.035E 02	1.066E 02	1.076E 02
J_r^{π}, l_r		$3/2^+$, 0	$3/2^+$, 0	1^+, 1	$5/2^-$, 2	$7/2^+$, 3	$5/2^+$, 1
ω_r	(64)	0.667	0.667	0.375	1.500	2.000	1.500
σ_r	(67)(77)	5.00	7.10 E−01	6.00 E−03	4.54 E−02	3.51 E−01	1.31 E−03
Γ_r	(67)	5.90 E−02	2.40 E−01	1.07 E−02	4.00 E−06	3.10 E−06	4.50 E−03
$(\omega\gamma)_r$	(64)(65)	1.04 E−02	2.23 E−02	8.29 E−06	4.50 E−08	5.50 E−07	3.50 E−06
S_r	(77)	1.94 E 01	1.19 E 01	2.05 E−01	1.62 E 06	9.15 E 04	1.49 E 02
$S(0)$	(76)	2.50	1.81	3.92 E−05	9.93 E−05	8.08 E−07	2.03 E−03
Coefficients in typical equations indicated in parentheses							
⟨01⟩	(82)	2.01 E−15	4.29 E−15	2.56 E−18	2.83 E−21	3.46 E−20	2.20 E−19
[01]	(78)	1.21 E 09	2.59 E 09	1.54 E 06	1.70 E 03	2.08 E 04	1.33 E 05
⟨23⟩	(83)	1.11 E−14	2.38 E−14	—	—	—	—
$[\lambda_\gamma]$	(80)(83)	6.69 E 09	1.43 E 10	1.01 E 17	6.85 E 13	8.37 E 14	5.32 E 15
$\tau_1(0)$	(79)	1.67 E−09	7.79 E−10	6.53 E−07	2.35 E−03	1.92 E−04	3.02 E−05
$\tau_0(1)$	(79)	2.50 E−09	1.17 E−09	4.55 E−06	4.12 E−03	3.37 E−04	5.30 E−05
$\mathcal{E}_{01}$	(81)	3.38 E 27	7.54 E 27	3.63 E 24	5.07 E 20	6.20 E 21	3.94 E 22
Uncertainty		+20% to −65%	+30% to −50%	±10%	±20%	±20%	±20%
T_9 limits		≥0.15	≥0.65	≥0.1	≥0.1	≥0.5	≥0.5
References		Ja 57	Ja 57	La 66	Aj 67	Aj 67	Aj 67

Reaction		$Be^7(p,\gamma)B^8$	$Be^7(\alpha,\gamma)C^{11}$	$Be^7(\alpha,\gamma)C^{11}$	$Be^9(p,d)Be^8$	$Be^9(p,d)Be^8$	$Be^9(p,d)Be^8$
Parameters in typical equations indicated in parentheses							
Q	(8)	0.135	7.545	7.545	0.559	0.559	0.559
E_x		0.778	8.105	8.427	6.884	7.433	8.071
E_r LAB		0.735	0.879	1.385	0.330	0.940	1.650
E_r CM	(64)(77)	0.643	0.560	0.882	0.297	0.845	1.484
$(E_r/k)_9$	(78)	7.462	6.499	1.024E 01	3.445	9.812	1.722E 01
E_r' CM	(68)	0.778	8.105	8.427	0.856	1.405	2.043
$(E_r'/k)_9$	(80)(83)	9.026	9.406E 01	9.780E 01	9.936	1.630E 01	2.371E 01
J_r^{π}, l_r		(1^+), (1)	$(3/2^-)$, (0)	$5/2^-$, 2	1^-, 0	2^-, 0	2^-, 0
ω_r	(64)	0.375	1.000	1.500	0.375	0.625	0.625
σ_r	(67)(77)	2.21 E−06	7.07 E−02	2.80 E−01	4.56 E−01	2.02 E−01	7.69 E−02
Γ_r	(67)	3.67 E−02	5.00 E−05	2.00 E−05	1.40 E−01	1.40 E−01	8.00 E−01
$(\omega\gamma)_r$	(64)(65)	1.75 E−08	1.92 E−06	4.80 E−06	6.53 E−03	8.23 E−03	3.15 E−02
S_r	(77)	1.46 E−04	8.52 E 05	1.72 E 05	1.36 E 02	1.03 E 01	2.51
$S(0)$	(76)	1.19 E−07	1.70 E−03	2.22 E−05	7.18	6.98 E−02	1.70 E−01
Coefficients in typical equations indicated in parentheses							
⟨01⟩	(82)	5.41 E−21	1.21 E−19	3.02 E−19	1.94 E−15	2.44 E−15	9.33 E−15
[01]	(78)	3.26 E 03	7.27 E 04	1.82 E 05	1.17 E 09	1.47 E 09	5.62 E 09
⟨23⟩	(83)	—	—	—	2.18 E−15	2.75 E−15	1.05 E−14
$[\lambda_\gamma]$	(80)(83)	4.25 E 13	2.92 E 15	7.30 E 15	1.31 E 09	1.66 E 09	6.33 E 09
$\tau_1(0)$	(79)	3.09 E−04	5.51 E−05	2.20 E−05	8.64 E−10	6.86 E−10	1.79 E−10
$\tau_0(1)$	(79)	2.15 E−03	9.65 E−05	3.86 E−05	7.73 E−09	6.13 E−09	1.60 E−09
$\mathcal{E}_{01}$	(81)	6.00 E 19	1.88 E 22	4.71 E 22	6.93 E 25	8.73 E 25	3.34 E 26
Uncertainty		±20%	FAC 3	FAC 3	±20%	±20%	±50%
T_9 limits		≥1	≥0.1	≥1	≥0.2	≥2	≥3
References		Pa 66, La 66	Aj 67	Aj 67	Mo 56	La 66	La 66

Units: Energy in MeV; σ_r in barns; S in MeV-barn; ⟨ ⟩ in $cm^3\ sec^{-1}$; [ ] in sec^{-1}; τ in sec; $\mathcal{E}_{01}$ in erg $g^{-1}\ sec^{-1}$; $T_9 = T/10^9$.
[a] Tabulated in La 66 as E_r LAB = 0.107 MeV. [b] Tabulated in La 66 as E_r LAB = 0.430 MeV.

TABLE V (*continued*)

CHARGED-PARTICLE RESONANT-REACTION DATA

Reaction		$Be^9(p,\alpha)Li^6$	$Be^9(p,\alpha)Li^6$	$Be^9(\alpha,n)C^{12}$	$Be^9(\alpha,n)C^{12}$	$Be^9(\alpha,n)C^{12}$	$Be^9(\alpha,n)C^{12}$
		Parameters in typical equations indicated in parentheses					
Q	(8)	2.126	2.126	5.704	5.704	5.704	5.704
E_x		6.884	7.433	11.011	11.066	11.980	12.209
E_r LAB		0.330	0.940	0.520	0.600	1.920	2.250
E_r CM	(64)(77)	0.297	0.845	0.360	0.415	1.330	1.558
$(E_r/k)_9$	(78)	3.445	9.812	4.179	4.822	1.543E 01	1.808E 01
E_r' CM	(68)	2.423	2.972	6.064	6.119	7.033	7.262
$(E_r'/k)_9$	(80)(83)	2.812E 01	3.449E 01	7.037E 01	7.101E 01	8.162E 01	8.427E 01
J_r^π, l_r		1^-, 0	2^-, 0	$(1/2^+)$, (1)	—	$(7/2^-)$, (2)	$(7/2^-)$, (2)
ω_r	(64)	0.375	0.625	0.500	—	2.000	2.000
σ_r	(67)(77)	3.52 E−01	2.49 E−01	1.80 E−04	—	2.90 E−01	1.75 E−01
Γ_r	(67)	1.40 E−01	1.40 E−01	5.54 E−02	—	1.38 E−01	2.77 E−01
$(\omega\gamma)_r$	(64)(65)	5.05 E−03	1.02 E−02	3.79 E−06	8.60 E−07	5.63 E−02	7.97 E−02
S_r	(77)	1.05 E 02	1.27 E 01	2.23 E 05	—	3.54 E 04	1.05 E 04
$S(0)$	(76)	5.55	8.64 E−02	1.31 E 03	—	9.58 E 01	8.21 E 01
		Coefficients in typical equations indicated in parentheses					
⟨01⟩	(82)	1.50 E−15	3.02 E−15	2.10 E−19	4.77 E−20	3.12 E−15	4.42 E−15
[01]	(78)	9.02 E 08	1.82 E 09	1.26 E 05	2.87 E 04	1.88 E 09	2.66 E 09
⟨23⟩	(83)	9.25 E−16	1.86 E−15	2.16 E−18	4.90 E−19	3.21 E−14	4.54 E−14
$[\lambda_\gamma]$	(80)(83)	5.57 E 08	1.12 E 09	1.30 E 06	2.95 E 05	1.93 E 10	2.74 E 10
$\tau_1(0)$	(79)	1.12 E−09	5.54 E−10	3.17 E−05	1.39 E−04	2.13 E−09	1.50 E−09
$\tau_0(1)$	(79)	9.99 E−09	4.96 E−09	7.13 E−05	3.14 E−04	4.79 E−09	3.39 E−09
$\mathcal{E}_{01}$	(81)	2.04 E 26	4.11 E 26	1.93 E 22	4.38 E 21	2.87 E 26	4.06 E 26
Uncertainty		±20%	±20%	±10%	±10%	±30%	±30%
T_9 limits		≥0.2	≥2	≥0.5	≥1	≥1	≥2
References		Mo 56	La 66	Da 66	Da 66	Gi 65	Gi 65

Reaction		$B^{10}(p,\alpha)Be^7$	$B^{10}(p,\alpha)Be^7$	$B^{11}(p,\gamma)C^{12}$	$B^{11}(p,\gamma)C^{12}$	$B^{11}(p,\gamma)C^{12}$	$B^{11}(p,2\alpha)He^4$
		Parameters in typical equations indicated in parentheses					
Q	(8)	1.148	1.148	15.957	15.957	15.957	8.682
E_x		9.756	10.086	16.106	16.575	17.228	16.106
E_r LAB		1.170	1.533	0.163	0.675	1.388	0.163
E_r CM	(64)(77)	1.063	1.393	0.149	0.618	1.272	0.149
$(E_r/k)_9$	(78)	1.234E 01	1.616E 01	1.733	7.177	1.476E 01	1.733
E_r' CM	(68)	2.211	2.540	16.106	16.575	17.228	8.832
$(E_r'/k)_9$	(80)(83)	2.565E 01	2.948E 01	1.869E 02	1.924E 02	1.999E 02	1.025E 02
J_r^π, l_r		$(5/2^+)$, (0)	$7/2^+$, 0	2^+, 1	2^-, 0	1^-, 0	2^+, 1
ω_r	(64)	0.429	0.571	0.625	0.625	0.375	0.625
σ_r	(67)(77)	2.05 E−01	3.00 E−01	1.58 E−04	5.00 E−05	5.30 E−05	1.02 E−02
Γ_r	(67)	2.73 E−01	2.27 E−01	6.41 E−03	2.95 E−01	1.16	6.41 E−03
$(\omega\gamma)_r$	(64)(65)	2.07 E−02	3.31 E−02	5.30 E−08	3.21 E−06	2.76 E−05	3.43 E−06
S_r	(77)	2.15 E 01	2.31 E 01	5.16	1.30 E−02	4.56 E−03	3.34 E 02
$S(0)$	(76)	3.47 E−01	1.52 E−01	2.38 E−03	7.01 E−04	7.89 E−04	1.54 E−01
		Coefficients in typical equations indicated in parentheses					
⟨01⟩	(82)	6.04 E−15	9.66 E−15	1.53 E−20	9.24 E−19	7.95 E−18	9.90 E−19
[01]	(78)	3.64 E 09	5.82 E 09	9.20 E 03	5.57 E 05	4.79 E 06	5.96 E 05
⟨23⟩	(83)	4.55 E−15	7.28 E−15	—	—	—	—
$[\lambda_\gamma]$	(80)(83)	2.74 E 09	4.38 E 09	6.46 E 14	3.90 E 16	3.36 E 17	2.09 E−04
$\tau_1(0)$	(79)	2.77 E−10	1.73 E−10	1.09 E−04	1.81 E−06	2.11 E−07	1.69 E−06
$\tau_0(1)$	(79)	2.75 E−09	1.72 E−09	1.20 E−03	1.98 E−05	2.30 E−06	1.85 E−05
$\mathcal{E}_{01}$	(81)	3.99 E 26	6.38 E 26	1.28 E 22	7.72 E 23	6.64 E 24	4.50 E 23
Uncertainty		±20%	±20%	±10%	±20%	±20%	±10%
T_9 limits		≥1	≥2	≥0.03	≥1	≥2	≥0.2
References		Aj 59	Aj 59	Aj 59	Aj 59	Aj 59	Aj 59

Units: Energy in MeV; σ_r in barns; S in MeV-barn; ⟨ ⟩ in $cm^3 sec^{-1}$; [ ] in sec^{-1}; τ in sec; $\mathcal{E}_{01}$ in erg $g^{-1} sec^{-1}$; $T_9 = T/10^9$.

TABLE V (*continued*)

CHARGED-PARTICLE RESONANT-REACTION DATA

Reaction		$B^{11}(p,2\alpha)He^4$	$B^{11}(p,2\alpha)He^4$	$C^{12}(p,\gamma)N^{13}$	$C^{13}(p,\gamma)N^{14}$	$C^{13}(p,\gamma)N^{14}$	$Ne^{20}(p,\gamma)Na^{21}$
Parameters in typical equations indicated in parentheses							
Q	(8)	8.682	8.682	1.944	7.550	7.550	2.432
E_x		16.575	17.228	2.368	8.065	8.710	2.810
E_r LAB		0.675	1.388	0.460	0.555	1.250	0.397
E_r CM	(64)(77)	0.618	1.272	0.424	0.515	1.160	0.378
$(E_r/k)_9$	(78)	7.177	1.476E 01	4.925	5.978	1.346E 01	4.381
E_r' CM	(68)	9.301	9.954	2.368	8.065	8.710	2.810
$(E_r'/k)_9$	(80)(83)	1.079E 02	1.155E 02	2.748E 01	9.359E 01	1.011E 02	3.261E 01
J_r^π, l_r		2^-, 0	1^-, 0	$1/2^+$, 0	1^-, 0	0^-, 0	$(1/2^+)$, (0)
ω_r	(64)	0.625	0.375	1.000	0.750	0.250	1.000
σ_r	(67)(77)	6.00 E−01	1.56 E−01	1.29 E−04	1.07 E−03	6.30 E−05	—
Γ_r	(67)	2.95 E−01	1.16	3.25 E−02	4.05 E−02	5.10 E−01	—
$(\omega\gamma)_r$	(64)(65)	3.85 E−02	8.11 E−02	6.29 E−07	7.94 E−06	1.33 E−05	1.00 E−06
S_r	(77)	1.56 E 02	1.34 E 01	3.58 E−01	1.64	1.51 E−02	—
$S(0)$	(76)	8.42	2.32	5.23 E−04	2.53 E−03	6.96 E−04	—
Coefficients in typical equations indicated in parentheses							
⟨01⟩	(82)	1.11 E−14	2.34 E−14	1.80 E−19	2.25 E−18	3.76 E−18	2.72 E−19
[01]	(78)	6.68 E 09	1.41 E 10	1.08 E 05	1.35 E 06	2.26 E 06	1.64 E 05
⟨23⟩	(83)	—	—	—	—	—	—
$[\lambda_\gamma]$	(80)(83)	2.34	4.93	9.56 E 14	1.61 E 16	2.69 E 16	7.60 E 14
$\tau_1(0)$	(79)	1.51 E−10	7.16 E−11	9.32 E−06	7.45 E−07	4.46 E−07	6.15 E−06
$\tau_0(1)$	(79)	1.65 E−09	7.82 E−10	1.11 E−04	9.62 E−06	5.75 E−06	1.22 E−04
$\mathcal{E}_{01}$	(81)	5.04 E 27	1.06 E 28	1.68 E 22	7.52 E 23	1.26 E 24	1.91 E 22
Uncertainty		±20%	±20%	−20% to +10%	−20% to +10%	+10% to −10%	FAC 3
T_9 limits		≥0.5	≥2	0.25 to 7	0.3 to 10	2.5 to 10	0.1 to 4
References		Aj 59	Aj 59	Vo 63	Vo 63	Se 52	En 62

Reaction		$N^{14}(p,\gamma)O^{15}$	$N^{14}(p,\gamma)O^{15}$	$N^{14}(p,\gamma)O^{15}$	$C^{13}(\alpha,n)O^{16}$	$C^{13}(\alpha,n)O^{16}$	$C^{13}(\alpha,n)O^{16}$
Parameters in typical equations indicated in parentheses							
Q	(8)	7.293	7.293	7.293	2.214	2.214	2.214
E_x		7.552	8.282	9.718	7.170	7.351	7.382
E_r LAB		0.278	1.060	2.600	1.063	1.300	1.340
E_r CM	(64)(77)	0.259	0.989	2.425	0.813	0.994	1.025
$(E_r/k)_9$	(78)	3.010	1.148E 01	2.815E 01	9.433	1.154E 01	1.189E 01
E_r' CM	(68)	7.552	8.282	9.718	3.027	3.208	3.239
$(E_r'/k)_9$	(80)(83)	8.764E 01	9.611E 01	1.128E 02	3.513E 01	3.723E 01	3.759E 01
J_r^π, l_r		$1/2^+$, 0	$3/2^+$, 0	—	5/2	—	5/2
ω_r	(64)	0.333	0.667	—	3.000	—	3.000
σ_r	(67)(77)	9.17 E−05	3.73 E−04	2.87 E−05	2.88 E−02	2.51 E−03	6.92 E−02
Γ_r	(67)	1.59 E−03	2.80 E−03	1.21	2.70 E−03	2.29 E−01	2.00 E−03
$(\omega\gamma)_r$	(64)(65)	1.35 E−08	3.69 E−07	3.01 E−05	7.37 E−05	6.68 E−04	1.65 E−04
S_r	(77)	1.27 E 01	3.15 E−01	5.20 E−03	2.37 E 08	2.78 E 06	5.77 E 07
$S(0)$	(76)	1.18 E−04	6.32 E−07	3.03 E−04	6.54 E 02	3.66 E 04	5.50 E 01
Coefficients in typical equations indicated in parentheses							
⟨01⟩	(82)	3.79 E−21	1.04 E−19	8.46 E−18	3.52 E−18	3.19 E−17	7.89 E−18
[01]	(78)	2.28 E 03	6.24 E 04	5.09 E 06	2.12 E 06	1.92 E 07	4.75 E 06
⟨23⟩	(83)	—	—	—	2.04 E−17	1.85 E−16	4.57 E−17
$[\lambda_\gamma]$	(80)(83)	6.16 E 13	1.68 E 15	1.37 E 17	1.23 E 07	1.11 E 08	2.75 E 07
$\tau_1(0)$	(79)	4.42 E−04	1.62 E−05	1.98 E−07	1.89 E−06	2.08 E−07	8.43 E−07
$\tau_0(1)$	(79)	6.14 E−03	2.24 E−04	2.75 E−06	6.14 E−06	6.77 E−07	2.74 E−06
$\mathcal{E}_{01}$	(81)	1.14 E 21	3.11 E 22	2.54 E 24	8.70 E 22	7.88 E 23	1.95 E 23
Uncertainty		±20%	±20%	±30%	±30%	±30%	±30%
T_9 limits		≥0.2	≥2	≥3	≥1	≥1	≥3
References		He 63	He 63	Ca 65	Wa 57	Wa 57	Wa 57

Units: Energy in MeV; σ_r in barns; S in MeV-barn; ⟨ ⟩ in cm³ sec⁻¹; [ ] in sec⁻¹; τ in sec; $\mathcal{E}_{01}$ in erg g⁻¹ sec⁻¹; $T_9 = T/10^9$.

TABLE V (*concluded*)

CHARGED-PARTICLE RESONANT-REACTION DATA

Reaction		$N^{15}(p,\gamma)O^{16}$	$N^{15}(p,\gamma)O^{16}$	$N^{15}(p,\alpha)C^{12}$	$N^{15}(p,\alpha)C^{12}$	$N^{15}(p,\alpha)C^{12}$	$O^{17}(p,\alpha)N^{14}$
		Parameters in typical equations indicated in parentheses					
Q	(8)	12.126	12.126	4.965	4.965	4.965	1.193
E_x		12.443	13.072	12.443	13.072	13.260	5.674
E_r LAB		0.338	1.010	0.338	1.010	1.210	0.069
E_r CM	(64)(77)	0.317	0.946	0.317	0.946	1.134	0.065
$(E_r/k)_9$	(78)	3.676	1.098*E* 01	3.676	1.098*E* 01	1.316*E* 01	0.754
E_r' CM	(68)	12.443	13.072	5.281	5.911	6.098	1.258
$(E_r'/k)_9$	(80)(83)	1.444*E* 02	1.517*E* 02	6.129*E* 01	6.860*E* 01	7.077*E* 01	1.460*E* 01
J_r^π, l_r		1^-,0	1^-,0	1^-,0	1^-,0	3^-,2	1^-,1
ω_r	(64)	0.750	0.750	0.750	0.750	1.750	0.250
σ_r	(67)(77)	6.91 *E*−06	9.83 *E*−04	7.66 *E*−02	4.58 *E*−01	6.87 *E*−01	7.51 *E*−09
Γ_r	(67)	8.82 *E*−02	1.31 *E*−01	8.82 *E*−02	1.31 *E*−01	2.11 *E*−02	2.68 *E*−04
$(\omega\gamma)_r$	(64)(65)	6.93 *E*−08	4.39 *E*−05	7.68 *E*−04	2.04 *E*−02	5.90 *E*−03	4.74 *E*−14
S_r	(77)	3.41 *E*−01	9.39 *E*−01	3.78 *E* 03	4.37 *E* 02	4.33 *E* 02	6.88 *E* 03
$S(0)$	(76)	6.48 *E*−03	4.49 *E*−03	7.19 *E* 01	2.09	3.74 *E*−02	2.92 *E*−02
		Coefficients in typical equations indicated in parentheses					
⟨01⟩	(82)	1.93 *E*−20	1.22 *E*−17	2.14 *E*−16	5.70 *E*−15	1.65 *E*−15	1.31 *E*−26
[01]	(78)	1.16 *E* 04	7.36 *E* 06	1.29 *E* 08	3.43 *E* 09	9.91 *E* 08	7.86 *E*−03
⟨23⟩	(83)	—	—	1.51 *E*−16	4.02 *E*−15	1.16 *E*−15	8.82 *E*−27
[λγ]	(80)(83)	4.22 *E* 14	2.67 *E* 17	9.11 *E* 07	2.42 *E* 09	7.00 *E* 08	5.31 *E*−03
$\tau_1(0)$	(79)	8.66 *E*−05	1.37 *E*−07	7.81 *E*−09	2.94 *E*−10	1.02 *E*−09	1.28 *E* 02
$\tau_0(1)$	(79)	1.29 *E*−03	2.04 *E*−06	1.16 *E*−07	4.37 *E*−09	1.51 *E*−08	2.16 *E* 03
$\mathcal{E}_{01}$	(81)	9.01 *E* 21	5.70 *E* 24	4.09 *E* 25	1.09 *E* 27	3.14 *E* 26	5.28 *E* 14
Uncertainty		±20%	±20%	±20%	±20%	±20%	±30%
T_9 limits		≥0.2	≥1	≥0.2	≥1	≥3	0.01 to 0.2
References		He 60	He 60	He 60	He 60	He 60	Br 62b

Reaction		$C^{12}(\alpha,\gamma)O^{16}$	$C^{12}(\alpha,\gamma)O^{16}$	$C^{12}(\alpha,\gamma)O^{16}$	$O^{16}(\alpha,\gamma)Ne^{20}$	$O^{16}(\alpha,\gamma)Ne^{20}$	$O^{16}(\alpha,\gamma)Ne^{20}$
		Parameters in typical equations indicated in parentheses					
Q	(8)	7.161	7.161	7.161	4.730	4.730	4.730
E_x		9.583	9.850	10.356	5.622	5.785	6.721
E_r LAB		3.229	3.585	4.260	1.116	1.319	2.490
E_r CM	(64)(77)	2.422	2.688	3.194	0.893	1.055	1.992
$(E_r/k)_9$	(78)	2.810*E* 01	3.120*E* 01	3.707*E* 01	1.036*E* 01	1.224*E* 01	2.311*E* 01
E_r' CM	(68)	9.583	9.850	10.356	5.622	5.785	6.721
$(E_r'/k)_9$	(80)(83)	1.112*E* 02	1.143*E* 02	1.202*E* 02	6.525*E* 01	6.713*E* 01	7.800*E* 01
J_r^π, l_r		1^-, 1	2^+, 2	4^+, 4	3^-, 3	1^-, 1	0^+, 0
ω_r	(64)	3.000	5.000	9.000	7.000	3.000	1.000
σ_r	(67)(77)	3.70 *E*−08	1.54 *E*−05	4.20 *E*−06	4.60 *E*−01	7.18 *E*−04	7.16 *E*−07
Γ_r	(67)	6.45 *E*−01	7.50 *E*−04	2.70 *E*−02	2.24 *E*−09	1.04 *E*−05	1.52 *E*−02
$(\omega\gamma)_r$	(64)(65)	6.60 *E*−08	3.55 *E*−08	4.14 *E*−07	1.12 *E*−09	9.60 *E*−09	2.64 *E*−08
S_r	(77)	4.92 *E*−02	1.16 *E* 01	1.34	4.29 *E* 12	7.16 *E* 08	7.41 *E* 02
$S(0)$	(76)	8.58 *E*−04	2.26 *E*−07	2.38 *E*−05	6.75 *E*−06	1.74 *E*−02	1.08 *E*−02
		Coefficients in typical equations indicated in parentheses					
⟨01⟩	(82)	3.25 *E*−21	1.75 *E*−21	2.04 *E*−20	5.00 *E*−23	4.29 *E*−22	1.18 *E*−21
[01]	(78)	1.96 *E* 03	1.05 *E* 03	1.23 *E* 04	3.01 *E* 01	2.58 *E* 02	7.10 *E* 02
⟨23⟩	(83)	—	—	—	—	—	—
[λγ]	(80)(83)	1.00 *E* 14	5.40 *E* 13	6.29 *E* 14	1.70 *E* 12	1.46 *E* 13	4.01 *E* 13
$\tau_1(0)$	(79)	2.05 *E*−03	3.81 *E*−03	3.26 *E*−04	1.33 *E*−01	1.55 *E*−02	5.64 *E*−03
$\tau_0(1)$	(79)	6.14 *E*−03	1.14 *E*−02	9.79 *E*−04	5.31 *E*−01	6.20 *E*−02	2.25 *E*−02
$\mathcal{E}_{01}$	(81)	2.81 *E* 20	1.51 *E* 20	1.76 *E* 21	2.15 *E* 18	1.84 *E* 19	5.06 *E* 19
Uncertainty		±30%	±20%	±20%	±30%	±30%	±30%
T_9 limits		2 to 6	2 to 6	2 to 6	≥0.2	≥0.3	≥3
References		La 64	La 64	La 64	Va 65	Va 65	Va 65

Units: Energy in MeV; σ_r in barns; S in MeV-barn; ⟨ ⟩ in cm³ sec⁻¹; [ ] in sec⁻¹; τ in sec; $\mathcal{E}_{01}$ in erg g⁻¹ sec⁻¹; $T_9=T/10^9$.

with J_r^π we list l_r, the minimum orbital angular momentum in units $\hbar$ with which the interacting nuclei 0+1 can form the resonant state in a manner consistent with the conservation of angular momentum and parity. When placed in parentheses the values tabulated are uncertain.

When used in Equations 62 to 64, the quantity $(\omega\gamma)_r$ must be evaluated in center-of-momentum coordinates. Thus, in the first form of Equation 67, Γ_r and E_r are the values in these coordinates as indicated. In actual measurements Γ_r and E_r are initially determined in laboratory coordinates in which case the second form of Equation 67 can be employed. Frequently the factor $[A_0/(A_0+A_1)]^3$ is taken as equal to unity. As measurements become more precise, we hope that this practice will be abandoned.

It will be clear from Equations 10, 16, 21, and 62 that the reaction rates for reverse reactions $\langle 23\rangle$, [23], $\lambda_\gamma(2)$, and [234] will contain an exponential factor with exponent $-E_r'/kT$ where

$$E_r' = Q + E_r \qquad 68.$$

is the resonance energy in center-of-momentum coordinates in the reverse system (2+3, 2+γ, or 2+3+4). The quantities E_r' CM and $(E_r'/k)_9$ are also listed in Tables IV and V.

It is frequently of interest to know the contribution made by the wing of a given resonance to the low-energy off-resonance region. In some cases the nonresonant reaction rate can be identified with the contribution from the wings of one or more specific resonances. An accurate calculation requires the use of the most sophisticated nuclear theory (Lane & Thomas 1958 or Humblet & Rosenfeld 1961), but a useful approximation for $E_r > E$ can be obtained through the use of Equation 61. It is assumed that $\Gamma_2 = \Gamma_2\ (Q+E)$ does not vary rapidly with the energy E of the interacting nuclei 0+1 on the basis that Q is positive for $0+1\rightarrow 2+3$ and is in general larger than E in the nonresonant range or at resonance E_r. Thus Γ_2 can be evaluated at resonance. The same assumption is made for any other partial width except Γ_1 which is small at low energies and in most cases does not make a contribution to Γ comparable to E_r. In other words we are essentially assuming that the resonance is sharp. Thus Γ can be evaluated at resonance Γ_r. With these approximations it is easily shown for s-wave ($l_r = 0$) neutron reactions that

$$\frac{\mathcal{S}(0)}{\mathcal{S}_r} = \frac{\Gamma_r^2/4}{E_r^2 + \Gamma_r^2/4} \qquad 69.$$

where

$$\mathcal{S}_r = \sigma_r v_r \qquad 70.$$

and

$$v_r = 1.383 \times 10^9\ (E_r\ \mathrm{LAB})^{1/2}\ \mathrm{cm\ sec^{-1}} \qquad 71.$$

For $l_r > 0$ the neutron partial width includes the factor E^{l_r} and thus $\mathcal{S}(0) = 0$ and at low energies $\mathcal{S}$ is negligible. Table IV lists v_r, $\mathcal{S}_r$, and $\mathcal{S}(0)$. An interesting example is the case of the reaction $N^{14}(n,p)C^{14}$. In this case the $l_r=0$ resonance with neutron laboratory energy equal to 0.639 MeV makes a contribution at low energy given approximately by $\mathcal{S}(0)=3.23\times10^{-19}$ cm^3 sec^{-1} while the accurately determined thermal-neutron cross section and velocity yield $\mathcal{S}(0)=3.98\times10^{-19}$ cm^3 sec^{-1}. To within the accuracy of our approximation the thermal-reaction rate can be attributed entirely to the low-energy wing of the 0.639-MeV resonance.

From Tables II and IV one has the following examples[6]

$$\begin{aligned}[He^3n]_p &= 6.0225\times10^{23}\,\rho\langle He^3n\rangle_p \\ &= 7.06\times10^8\,\rho(1-0.597\,T_9^{1/2}+0.183\,T_9)\quad 0\le T_9\le 10 \\ &\quad +1.29\times10^{11}\,\rho T_9^{-3/2}\exp(-20.61/T_9)\ \text{sec}^{-1}\quad 3\le T_9\le 10\end{aligned} \tag{72.}$$

$$\begin{aligned}[pT^3]_n &= 6.0225\times10^{23}\,\rho\langle pT^3\rangle_n \\ &= 7.07\times10^8\,\rho(1-0.597\,T_9^{1/2}+0.183\,T_9)\exp(-8.864/T_9)\quad 0\le T_9\le 10 \\ &\quad +1.29\times10^{11}\,\rho T_9^{-3/2}\exp(-29.47/T_9)\ \text{sec}^{-1}\quad 3\le T_9\le 10\end{aligned} \tag{73.}$$

$$\begin{aligned}\frac{1}{\tau_{np}(He^3)} &= \frac{1}{1.43\times10^{-9}}\,\rho X_n\,(1-0.597\,T_9^{1/2}+0.183\,T_9)\quad 0\le T_9\le 10 \\ &\quad +\frac{1}{7.81\times10^{-12}}\,\rho X_n T_9^{-3/2}\exp(-20.61/T_9)\ \text{sec}^{-1}\quad 3\le T_9\le 10\end{aligned} \tag{74.}$$

$$\begin{aligned}\mathcal{E}(He^3n)_p &= 1.71\times10^{26}\,\rho X_n X_{He^3}(1-0.597\,T_9^{1/2}+0.183\,T_9)\quad 0\le T_9\le 10 \\ &\quad +3.13\times10^{28}\,\rho X_n X_{He^3}T_9^{-3/2}\exp(-20.61/T_9)\ \text{erg g}^{-1}\,\text{sec}^{-1}\quad 3\le T_9\le 10\end{aligned} \tag{75.}$$

For charged particles the equations corresponding to 69 and 70 are

$$\frac{S(0)}{S_r} = \frac{\Gamma_r^2/4}{E_r^2+\Gamma_r^2/4} \tag{76.}$$

where

$$S_r = \sigma_r E_r \exp(E_G/E_r)^{1/2} \tag{77.}$$

These expressions hold for any partial wave in the case of charged particles. Table V lists S_r and $S(0)$. In addition to the approximation inherent in using Equation 61, we have neglected any variation in S other than the simple resonance behavior exhibited in Equation 76. In particular we have neglected variations in the penetration factor other than the term in $\exp-(E_G/E)^{1/2}$ used in Equation 45. For resonances at energies greater than 20 per cent of the Coulomb-barrier energy this results in a value for $S(0)$ given by Equation 76 which could be low by as much as an order of magnitude. In most cases independent measurements on S are available in the low-energy range and it is not necessary to calculate $S(0)$ from resonance data.

There is an additional point of considerable importance: the amplitude

[6] When the nonresonant term is available, it is given first in all equations. The additional terms for one or more resonances then follow. In using the references to numbered equations in the tables, use the nonresonant terms for Tables II and III and the appropriate resonant terms for Tables IV and V.

contributions of states of the same spin and parity must be summed coherently. In general the phase is zero or π between two fairly well-separated states so that the overall contribution to $S(0)$ is the square of the sum or difference of the square roots of the $S(0)$ for the two states. In the case of $N^{15}(p,\gamma)O^{16}$ the resonances at laboratory proton energies 0.338 MeV and 1.010 MeV both have $J^\pi = 1^-$ and are known to interfere constructively at low energies. With the data for $S(0)$ in Table V our method of calculation yields $S(0) = [(6.48 \times 10^{-3})^{1/2} + (4.49 \times 10^{-3})^{1/2}]^2 = 2.18 \times 10^{-2}$ MeV-barn. A more sophisticated treatment of the problem by Hebbard (1960) (see also Caughlan & Fowler 1962) yields $S(0) = 2.74 \times 10^{-2}$ MeV-barn as given in Table III. In the case of $N^{15}(p,\alpha)C^{12}$ the two states interfere destructively, yielding $S(0) = [(71.9)^{1/2} - (2.09)^{1/2}]^2 = 49.5$ MeV-barn compared to the more accurate value, 53.4 MeV-barn (Hebbard 1960), given in Table III. It is also possible to calculate resonance contributions to $S'(0)$ by similar methods and this was done by Caughlan & Fowler (1962) using the data of Hebbard (1960) to give the values for $N^{15}(p, \gamma)O^{16}$ and $N^{15}(p,\alpha)C^{12}$ given in Table III.

The $C^{12}(p,\gamma)N^{13}$ *and* $C^{13}(p,\gamma)N^{14}$ *reactions.*—These reactions have been chosen to illustrate in detail the use of the charged-particle nonresonant and resonant data given in Tables III and V. The experimentally determined cross section for $C^{12}(p,\gamma)N^{13}$ is shown in Figure 1 from the work of Hebbard & Vogl (1960) and Vogl (1963). Earlier sources are cited in these references. The cross section for $C^{13}(p,\gamma)N^{14}$ is shown in Figure 2 and has been taken from Seagrave (1952) as well as from the two references just cited. The measurements of Seagrave on $C^{13}(p,\gamma)N^{14}$ have been adjusted to those of Vogl in the region of energy overlap. The solid curves have been calculated by the present authors with the full apparatus of the R-matrix theory described by Lane & Thomas (1958) using only the empirical parameters which can be determined at or near the observed resonances. In the case of the 0.460-MeV resonance in $C^{12}(p,\gamma)$, the Thomas (1952) factor has been used. Similar success in fitting the data has been achieved by Mahaux (1965) using the theory of Humblet & Rosenfeld (1961).

The extrapolations of our calculated curves through the low-energy experimental data are shown in Figure 3. It will be clear that the $C^{12}(p,\gamma)N^{13}$ reaction can be accounted for entirely in terms of the properties of the 0.460-MeV state ($E_x = 2.368$ MeV in N^{14}) over the energy range of interest. Similarly, with the exception of the regions near the sharp resonances in Figure 2, the $C^{13}(p,\gamma)N^{14}$ reaction is accounted for in terms of the properties of the relatively wide 0.555-MeV and 1.250-MeV resonances over the entire energy range from 0 to 1.5 MeV. These two resonances have $J^\pi = 1^-$ and 0^- respectively, so their cross-section contributions simply add at low energies.

The calculated curves given in Figures 1 and 2 are not expressible in terms of simple analytic formulae. However, it is possible in the case of these reactions to make a fairly accurate determination of the parameters entering into the analytic expressions for nonresonant and resonant cross sections under

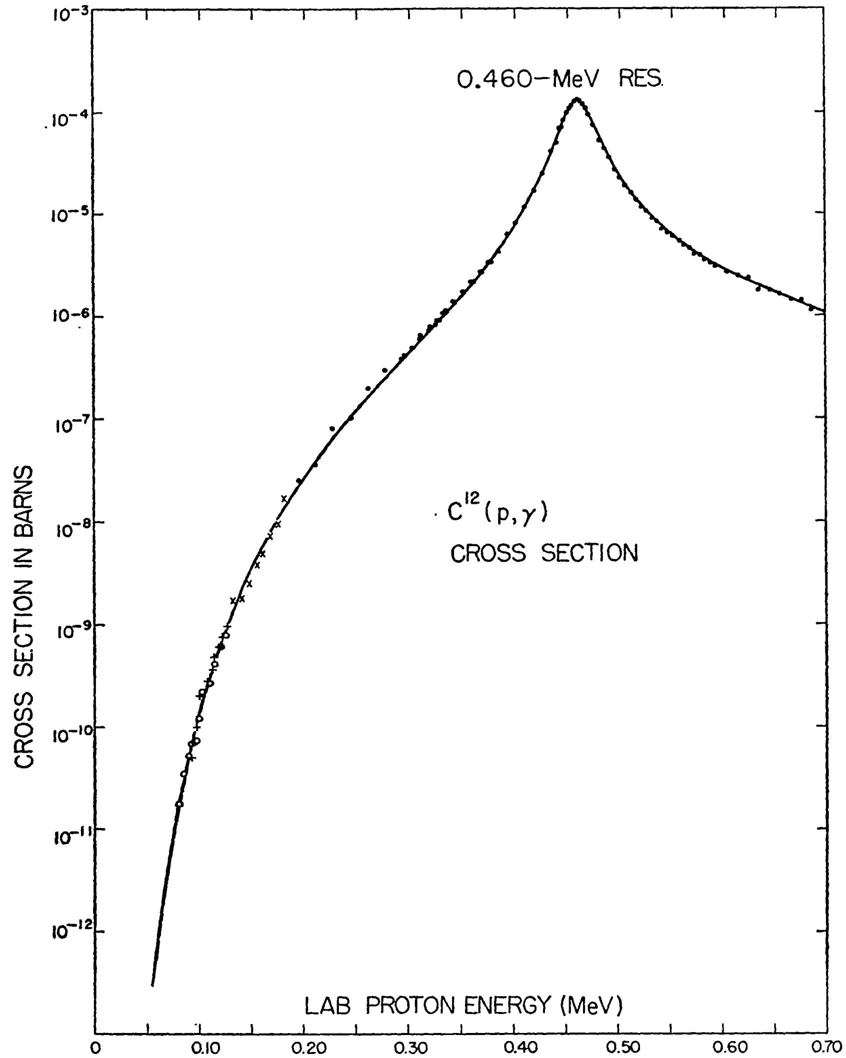

FIG. 1. The dependence on energy of the cross section for the reaction $C^{12}(p,\gamma)N^{13}$. Sources of the data are cited in the text. The solid curve is a theoretical curve based on the R-matrix theory described by Lane & Thomas (1958) using empirical parameters determined at or near the 0.460-MeV resonance.

discussion in this paper. We have carried out the necessary numerical integrations over the observed cross sections to determine $\langle\sigma v\rangle$ as a function of temperature. This was followed by a least-squares analysis using Equations 50 and 52 to determine $S(0)$, $S'(0)/S(0)$, and $\frac{1}{2}S''(0)/S(0)$ for both $C^{12}(p,\gamma)N^{13}$ and $C^{13}(p,\gamma)N^{14}$. Similarly a least-squares analysis was made to determine E_r, σ_r, and Γ_r for the 0.460-MeV, 0.555-MeV, and 1.250-MeV resonances. The percent deviation of the final analytical expressions given in Equations 78 and 82 below from the integrated values is shown as a function of temperature in Figure 4. By judicious choice of the temperature limits on the nonresonant and resonant expressions it has been found possible to keep the errors below ~20 per cent. The deviation-curve for $C^{12}(p,\gamma)N^{13}$ in Figure 4 illustrates the error which arises in using the approximate resonance expression given in Equation 62 for broad resonances. Note that the analytic expression falls below the actual integrated value on either side of the temperature range within which E_r is approximately equal to $E_0(T)$.

All of the numerical parameters have been recorded in Tables III and V and we now employ these to illustrate the use of the tables in forming desired analytic expressions:

$$\begin{aligned}[C^{12}p]_\gamma &= 6.0225\times10^{23}\,\rho\langle C^{12}p\rangle_\gamma \\ &= 2.04\times10^{7}\,(1+0.0304\,T_9^{1/3}+1.19\,T_9^{2/3}+0.254\,T_9+2.06\,T_9^{4/3} \\ &\quad +1.12\,T_9^{5/3})\,\rho\,T_9^{-2/3}\exp(-13.69/T_9^{1/3}) \qquad 0\le T_9\le 0.55 \\ &\quad +1.08\times10^{5}\,\rho\,T_9^{-3/2}\exp(-4.925/T_9)\ \text{sec}^{-1} \qquad 0.25\le T_9\le 7\end{aligned} \quad 78.$$

$$\begin{aligned}\frac{1}{\tau_{p\gamma}(C^{12})} &= \frac{1}{4.93\times10^{-8}}(1+0.0304\,T_9^{1/3}+1.19\,T_9^{2/3}+0.254\,T_9+2.06\,T_9^{4/3} \\ &\quad +1.12\,T_9^{5/3})\,\rho X_H T_9^{-2/3}\exp(-13.69/T_9^{1/3}) \qquad 0\le T_9\le 0.55 \\ &\quad +\frac{1}{9.32\times10^{-6}}\rho X_H T_9^{-3/2}\exp(-4.925/T_9)\ \text{sec}^{-1} \qquad 0.25\le T_9\le 7\end{aligned} \quad 79.[7]$$

$$\begin{aligned}\lambda_{\gamma p}(N^{13}) &= 1.81\times10^{17}\,(1+0.0304\,T_9^{1/3}+1.19\,T_9^{2/3}+0.254\,T_9+2.06\,T_9^{4/3} \\ &\quad +1.12\,T_9^{5/3})\,T_9^{5/6}\exp(-13.69/T_9^{1/3}-22.56/T_9) \qquad 0\le T_9\le 0.55 \\ &\quad +9.56\times10^{14}\exp(-27.48/T_9)\ \text{sec}^{-1} \qquad 0.25\le T_9\le 7\end{aligned} \quad 80.$$

$$\begin{aligned}\mathcal{E}(C^{12}p)_\gamma &= 3.17\times10^{24}\,(1+0.0304\,T_9^{1/3}+1.19\,T_9^{2/3}+0.254\,T_9+2.06\,T_9^{4/3} \\ &\quad +1.12\,T_9^{5/3})\,\rho X_H X_{C^{12}}\,T_9^{-2/3}\exp(-13.69/T_9^{1/3}) \qquad 0\le T_9\le 0.55 \\ &\quad +1.68\times10^{22}\,\rho X_H X_{C^{12}}\,T_9^{-3/2}\exp(-4.925/T_9) \qquad 0.25\le T_9\le 7 \\ &\quad \text{erg g}^{-1}\ \text{sec}^{-1}\end{aligned} \quad 81.$$

$$\begin{aligned}\langle C^{13}p\rangle_\gamma &= 1.6604\times10^{-24}\,\rho^{-1}[C^{13}p]_\gamma \\ &= 1.33\times10^{-16}\,(1+0.0304\,T_9^{1/3}+0.958\,T_9^{2/3}+0.204\,T_9+1.39\,T_9^{4/3} \\ &\quad +0.753\,T_9^{5/3})\,T_9^{-2/3}\exp(-13.72/T_9^{1/3}) \qquad 0\le T_9\le 0.65 \\ &\quad +2.25\times10^{-18}\,T_9^{-3/2}\exp(-5.978/T_9) \qquad 0.3\le T_9\le 10 \\ &\quad +3.76\times10^{-18}\,T_9^{-3/2}\exp(-13.46/T_9)\ \text{cm}^3\ \text{sec}^{-1} \qquad 2.5\le T_9\le 10\end{aligned} \quad 82.$$

etc. etc.

It is necessary to emphasize that our procedures in determining the resonance parameters E_r, Γ_r, and $(\omega\gamma)_r$ from fitting $\langle\sigma v\rangle$ as a function of temperature do not necessarily result in agreement with the usual values obtained by fitting σ as a function of energy. This is notably true in the cases just discussed in detail and in the cases $T(d,n)He^4$ and $He^3(d,p)He^4$. In general, resonances which are not sharp are not accurately treated by use of the approximation inherent in Equation 62. We find, however, that some adjust-

[7] Note that the coefficients listed in the tables occur here in the denominator for $1/\tau$ and are thus the correct coefficients in the numerator for τ when only one term is employed.

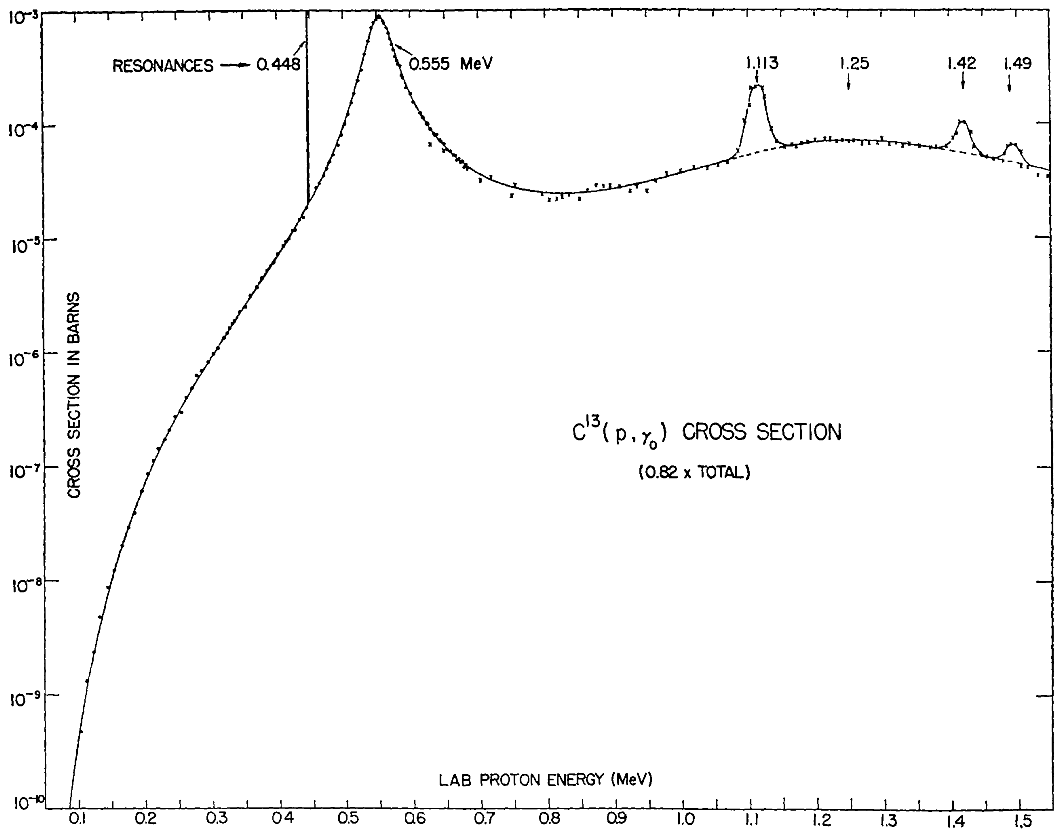

FIG. 2. The dependence on energy of the cross section for the reaction $C^{13}(p,\gamma_0)N^{14}$, where γ_0 designates the gamma-ray transition to the ground state of N^{14}. This transition accounts for 82 per cent of the total cross section which includes cascade transitions through excited states of N^{14}. Sources of the data are cited in the text. The solid curve, exclusive of the resonant features at 0.448, 1.113, 1.42, and 1.49 MeV, is a theoretical curve based on the R-matrix theory described by Lane & Thomas (1958) using empirical parameters determined at or near the 0.555-MeV and 1.250-MeV resonances.

ment in the empirical parameters as just described results in a tolerable uncertainty in the analytic expression for the reaction rate.

Examples of reverse reactions.—The reaction rates for $Be^7(\alpha,p)B^{10}$ which is the reverse of $B^{10}(p,\alpha)Be^7$ and for $He^4(2p,\tau)He^3$ which is the reverse of $He^3(\tau,2p)He^4$ can be found from

$$\begin{aligned}[\alpha Be^7]_p &= 6.0225 \times 10^{23}\,\rho\langle\alpha Be^7\rangle_p \\ &= 9.48 \times 10^{10}\,(1 + 0.0345\,T_9^{1/3} - 0.498\,T_9^{2/3} - 0.121\,T_9 + 0.300\,T_9^{4/3} \\ &\quad + 0.185\,T_9^{5/3})\,\rho\,T_9^{-2/3}\exp(-12.06/T_9^{1/3} - 13.32/T_9) \qquad 0 \le T_9 \le 5 \\ &\quad + 2.74 \times 10^{9}\,\rho T_9^{-3/2}\exp(-25.65/T_9) \qquad 1 \le T_9 \le 10 \\ &\quad + 4.38 \times 10^{9}\,\rho T_9^{-3/2}\exp(-29.48/T_9)\ \text{sec}^{-1} \qquad 2 \le T_9 \le 10\end{aligned} \qquad 83.$$

$$[2pHe^4]_\tau = 18.4\,\rho^2 T_9^{-13/6}\exp(-12.28/T_9^{1/3} - 149.2/T_9)\ \text{sec}^{-1} \qquad 0 \le T_9 \le 10 \qquad 84.$$

The $3He^4 \rightarrow C^{12} + 7.274$*-MeV process.*—This process is a special case and must be given special consideration. The conversion of three alpha particles into a C^{12} nucleus takes place in two resonant stages and can be written in detailed notation as $He^4(\alpha)Be^8(\alpha)C^{12*}(\gamma\gamma$ or $e^+e^-)C^{12}$. The ground state of Be^8 serves as one resonance and either the 7.644-MeV or the 9.638-MeV excited state in C^{12} as the other. The ground state of Be^8 is unbound with respect to decay into two alpha particles by 0.092 MeV according to very precise measurements by Benn, Dally, Müller, Pixley, Staub & Winkler (1966). The energy of the 7.644-MeV state in C^{12} is calculated using C^{12}-Be^8-$He^4 = 0.278$

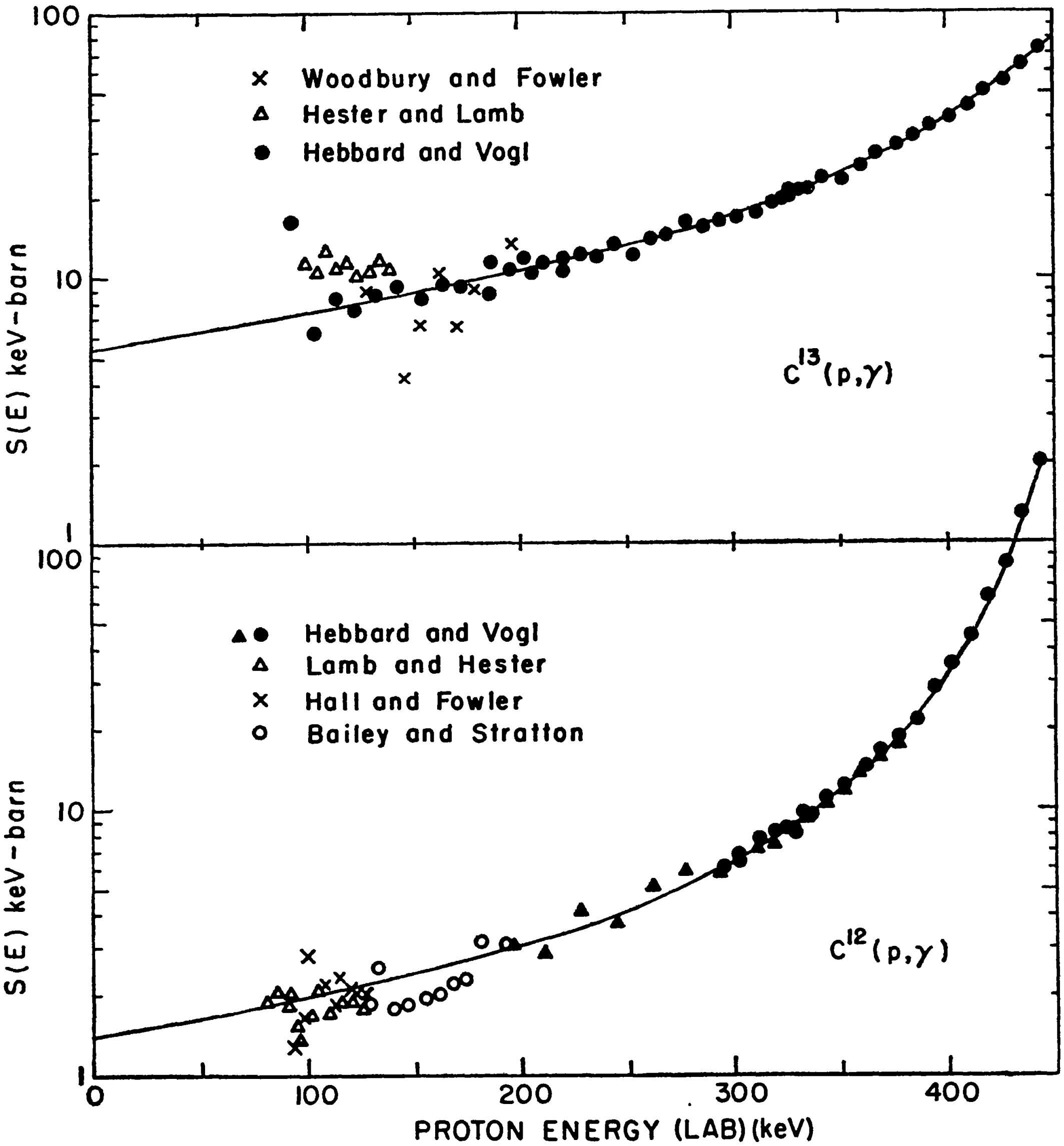

FIG. 3. The dependence on energy of the cross-section factors for the reactions $C^{12}(p,\gamma)N^{13}$ and $C^{13}(p,\gamma)N^{14}$.

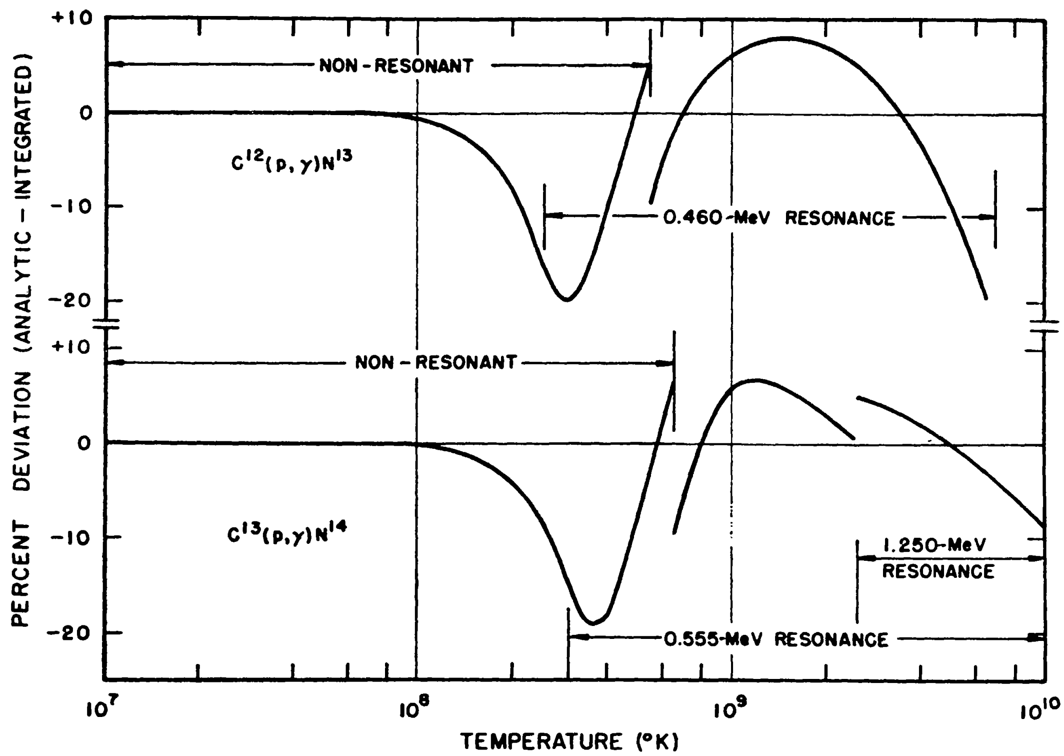

FIG. 4. *Upper curve:* The percent deviation of the analytical expressions for $\langle\sigma v\rangle$ for $C^{12}(p,\gamma)N^{13}$ given by Equation 78 from the values given by actual integration over the cross section illustrated in Figure 1.

Lower curve: The percent deviation of the analytical expression for $\langle\sigma v\rangle$ for $C^{13}(p,\gamma)N^{14}$ given by Equation 82 from the values given by actual integration over the cross section illustrated in Figure 2.

MeV/c^2 as determined by Cook, Fowler, Lauritsen & Lauritsen (1957). The 7.644-MeV state cascades by gamma-ray emission through the excited state in C^{12} at 4.433 MeV and also emits an electron-positron pair directly to the ground state. The 9.638-MeV state most probably cascades by an electric dipole transition through the 4.433-MeV state. This transition is inhibited by an isotopic-spin selection rule. Detailed measurements on the 7.644-MeV state are summarized in Seeger & Kavanagh (1963). The pair-emission width of the 7.644-MeV state enters directly into the determination of the total radiation width Γ_{rad} for this state. Seeger & Kavanagh used $(5.5 \pm 3) \times 10^{-11}$ MeV. Recent determinations are $(6.5 \pm 0.7) \times 10^{-11}$ MeV (Crannell & Griffy 1964), $(6.2 \pm 0.5) \times 10^{-11}$ MeV (Crannell 1965), and $(7.3 \pm 1.3) \times 10^{-11}$ MeV (Gudden & Strehl 1965). We take $(6.5 \pm 0.6) \times 10^{-11}$ MeV and find $\Gamma_{rad} = (2.8 \pm 0.5) \times 10^{-9}$ MeV. The statistical weight factor is $\omega = 2J+1 = 1$ since the state has $J^\pi = 0^+$. For the 9.638-MeV state, $J^\pi = 3^-$ and $\omega = 7$. On the basis of the isotopic-spin selection rule, Hoyle & Fowler (1960) estimate $\Gamma_\gamma \sim 10^{-8}$ MeV (4 per cent of the full electric dipole width) so that $\omega\Gamma_\gamma \sim 7 \times 10^{-8}$ MeV. The excess energies of the two excited states in C^{12} over $3He^4$ are 0.370 MeV and 2.364 MeV respectively. With these data the rate of the $3\alpha \rightarrow C^{12}$ process is found to be

$$P_{3\alpha} = \tfrac{1}{6} n_{He^4}^3 \langle \alpha\alpha\alpha \rangle = 1.565 \times 10^{21}\, \rho X_{He^4}^3 [\alpha\alpha\alpha] \text{ reactions cm}^{-3} \text{ sec}^{-1} \qquad 85.$$

while the lifetime of He^4 to the process is given by

$$\frac{1}{\tau_{3\alpha}(He^4)} = -\frac{1}{n_{He^4}} dn_{He^4}/dt = -\frac{1}{X_{He^4}} dX_{He^4}/dt = \frac{1}{X_{He^4}} dX_{C^{12}}/dt$$

$$= 3P_{3\alpha}/n_{He^4} = \tfrac{1}{2} n_{He^4}^2 \langle \alpha\alpha\alpha \rangle = \frac{1}{32.04} X_{He^4}^2 [\alpha\alpha\alpha] \qquad 86.$$

where

$$\langle \alpha\alpha\alpha \rangle = 5.861 \times 10^{-56}\, T_9^{-3} \exp(-4.294/T_9) \qquad 0.03 \leq T_9 \leq 8$$
$$+ 1.465 \times 10^{-54}\, T_9^{-3} \exp(-27.433/T_9) \text{ cm}^6 \text{ sec}^{-1} \qquad 4 \leq T_9 \leq 8 \qquad 87.$$

$$[\alpha\alpha\alpha] = \rho^2 N_A^2 \langle \alpha\alpha\alpha \rangle = 3.627 \times 10^{47} \rho^2 \langle \alpha\alpha\alpha \rangle$$
$$= 2.126 \times 10^{-8} \rho^2 T_9^{-3} \exp(-4.294/T_9) \qquad 0.03 \leq T_9 \leq 8 \qquad 87'.$$
$$+ 5.315 \times 10^{-7} \rho^2 T_9^{-3} \exp(-27.433/T_9) \text{ sec}^{-1} \qquad 4 \leq T_9 \leq 8$$

The photodisintegration rate for C^{12} becomes

$$\frac{1}{\tau_{\gamma\alpha}(C^{12})} = \lambda_{\gamma\alpha}(C^{12}) = 4.257 \times 10^{12} \exp(-88.71/T_9) \qquad 0.03 \leq T_9 \leq 8$$
$$+ 1.064 \times 10^{14} \exp(-111.85/T_9) \text{ sec}^{-1} \qquad 4 \leq T_9 \leq 8 \qquad 88.$$

The energy generation rate is

$$\mathcal{E}(3He^4 \rightarrow C^{12}) = 3.88 \times 10^8\, \rho^2 X_{He^4}^3 T_9^{-3} \exp(-4.294/T_9) \qquad 0.03 \leq T_9 \leq 8$$
$$+ 9.70 \times 10^9\, \rho^2 X_{He^4}^3 T_9^{-3} \exp(-27.433/T_9) \qquad 4 \leq T_9 \leq 8 \qquad 89.$$
$$\text{erg g}^{-1} \text{ sec}^{-1}$$

The 3 in the numerator of the first term on the right-hand side of Equation 86 occurs because three He^4 nuclei are destroyed in each reaction. The 6 in the denominator of Equation 85 arises from the identity of the three alpha particles. Equation 88 assumes that the 4.433-MeV state in C^{12} is in equilibrium with the ground state. The uncertainty in the above equations is approximately ± 60 per cent for the term due to the 7.644-MeV state and a factor of at least 3 for the term due to the 9.638-MeV state. Fortunately, in the applications currently thought to be of importance, the contribution of the 9.638-MeV state can be neglected.

The $C^{12}(\alpha,\gamma)O^{16}+7.161$-MeV reaction.—This reaction is also a special case and must be given special consideration. At low energy the cross section is dominated by the contribution of the 1^- excited state of O^{16} at 7.115 MeV which is bound by $|E_r| = 0.046$ MeV. The partial widths are small compared to $|E_r|$, and the term $\Gamma^2/4$ in the resonance denominator of Equation 61 can be neglected. In addition, in determining $\langle \sigma v \rangle$ from σ it is sufficiently accurate to substitute E_0 for E in the resonance denominator of this equation. This procedure was employed by B^2FH (1957), which should be consulted for additional details. The result which was obtained can be expressed as

$$S_{eff} = 2.67 \times 10^6 \frac{\theta_\alpha^2 \Gamma_\gamma}{(E_0 + |E_r|)^2} \text{ MeV-barn}$$
$$= 3.14 \times 10^6 \frac{\theta_\alpha^2 \Gamma_\gamma}{T_9^{4/3}(1 + 0.050\, T_9^{-2/3})^2} \qquad 90.$$
$$= \frac{1.76 \times 10^{-2}}{T_9^{4/3}(1 + 0.050\, T_9^{-2/3})^2}$$

where θ_α^2 is the dimensionless, reduced alpha-particle width for the 7.12-MeV excited state and Γ_γ is the radiation width in MeV. The final numerical result has been obtained by using $E_0 = 0.9226\, T_9^{2/3}$ MeV, $\Gamma_\gamma = 6.6 \times 10^{-8}$ MeV from the measurements of Swann & Metzger (1957), and $\theta_\alpha^2(7.12) = (0.1 \pm 0.05) \times \theta_\alpha^2(9.58) = 0.085 \pm 0.040$ from a recent theoretical calculation by Stephenson (1966). The reduced alpha-particle width for the 9.58-MeV excited state of O^{16} has been measured to be equal to 0.85 (Ajzenberg-Selove & Lauritsen 1959). Stephenson's calculation employs a specific model for the O^{16} nucleus but his result has been confirmed by indirect measurements by Loebenstein, Mingay, Winkler & Zaidins (1967) who find the result $0.06 \leq \theta_\alpha^2 \leq 0.14$.

When Equation 90 is substituted into Equation 51 one obtains

$$[C^{12}\alpha]_\gamma = 6.0225 \times 10^{23} \rho \langle C^{12}\alpha \rangle_\gamma \qquad 91.$$
$$= 2.19 \times 10^8 \rho T_9^{-2}(1 + 0.050\, T_9^{-2/3})^{-2} \exp(-32.12/T_9^{1/3}) \text{ sec}^{-1}$$
$$\text{nonres,} \quad 0 \leq T_9 \leq 6$$

so that

$$\frac{1}{\tau_{\alpha\gamma}(C^{12})} = \frac{1}{1.83 \times 10^{-8}} \rho X_{He^4} T_9^{-2}(1 + 0.050\, T_9^{-2/3})^{-2} \exp(-32.12/T_9^{1/3}) \text{ sec}^{-1} \qquad 92.$$
$$\text{nonres,} \quad 0 \leq T_9 \leq 6$$

$$\lambda_{\gamma\alpha}(O^{16}) = 1.12 \times 10^{19}\, T_9^{-1/2}(1 + 0.050\, T_9^{-2/3})^{-2} \times \exp(-32.12/T_9^{1/3} - 83.11/T_9) \text{ sec}^{-1} \quad \text{nonres,} \quad 0 \leq T_9 \leq 6 \qquad 93.$$

and

$$\mathcal{E}(C^{12}\alpha)_\gamma = 3.15 \times 10^{25} \rho X_{He^4} X_{C^{12}} T_9^{-2}(1 + 0.050\, T_9^{-2/3})^{-2} \times \exp(-32.12/T_9^{1/3}) \text{ erg g}^{-1} \text{ sec}^{-1} \quad \text{nonres,} \quad 0 \leq T_9 \leq 6 \qquad 94.$$

The uncertainty in the numerical coefficients in the above expressions is ± 50 per cent. At high temperatures the resonances in $C^{12}(\alpha,\gamma)O^{16}$ listed in Table V make substantial contributions and *these contributions must be added to the nonresonant terms given in Equations 91 to 94*. Data on subsequent helium-burning reactions $O^{16}(\alpha,\gamma)Ne^{20}$ and $Ne^{20}(\alpha,\gamma)Mg^{24}$ are given in Tables III, V, and VI (to be discussed in what follows).

Continuum-reaction rates.—In this section the interactions of protons, alpha particles, and photons with nuclei in the mass range $19 \leq A \leq 40$ are discussed. These interactions occur under astrophysical circumstances at temperatures in excess of 10^9 °K and thus at effective energies in excess of the order of 1 MeV. Under these circumstances numerous resonances contribute to the reaction cross section, and in some cases these resonances are broad enough to overlap and produce a continuum cross section characterized by broad maxima and minima. Even in the case of sharp, well-isolated resonances, the great breadth of the effective range in interaction energy means that the smoothed-out cross section obtained by averaging over all resonances within the appropriate energy range is of primary interest. In any case it is a simple computational matter to calculate the total $\langle \sigma v \rangle$ by summing over the contributions of individual resonances as given by Equation 62. This is facilitated by the fact that most authors express their experimental results in terms of the quantities E_r and $(\omega\gamma)_r$ required to evaluate Equation 62.

We have carried out such a computational program to obtain $\langle \sigma v \rangle$ as a function of temperature over the range $1 \leq T_9 \leq 5$ for the reactions listed in Table VI. We have supplemented the data given in Ajzenberg-Selove & Lauritsen (1959) and Endt & Van der Leun (1962) by a fairly extensive search of the recent literature. A least-squares analysis has been made in each case of the fit of $\langle \sigma v \rangle$ versus T to equations of the form $T^n \exp(-E_{\text{th}}/kT)$, where E_{th} is an effective "threshold" energy and $n = -\frac{1}{2}$, 0, $\frac{1}{2}$, and $\frac{3}{2}$. Our analyses showed that in almost all cases the best fit was obtained by the use of $n = 0$, and because of the inherent simplicity of the resulting equation we have, in all cases, used

$$[01] = \rho N_A \langle 01 \rangle = C\rho \exp(-E_{\text{th}}/kT) \qquad 95.$$

where C as well as E_{th} are semi-empirical constants determined by a least-squares analysis.

The threshold energy is that energy above which barrier penetration effects in Γ_1 are no longer of importance in determining $(\omega\gamma)_r$ as defined by Equation 65. This always occurs as the energy approaches the top of the Coulomb barrier, but it also occurs in the (p,γ), (p,α), and (α,γ) reactions of interest here at considerably lower energy. In all these reactions Γ_1 rises rapidly until it is the major partial width in $\Gamma = \Gamma_1 + \Gamma_2 + \ldots$ so that $\Gamma \approx \Gamma_1$ and $(\omega\gamma)_r \approx (\omega\Gamma_2)_r$, where Γ_2 varies from resonance to resonance but shows no systematic rapid increase or decrease with the energy of the interacting particles. The corresponding threshold energy in the reverse reaction $2+3 \rightarrow 0+1$ is given by

$$E_{\text{th}}' = E_{\text{th}} + Q \qquad 96.$$

Our results are tabulated in Table VI which yields, as examples, the following results for $Ne^{20}(\alpha,\gamma)Mg^{24}$, $Mg^{24}(\gamma,\alpha)Ne^{20}$, and $Ne^{20}(\alpha,p)Na^{23}$:

$$\begin{aligned}[Ne^{20}\alpha]_\gamma &= 6.0225 \times 10^{23}\, \rho \langle Ne^{20}\alpha \rangle_\gamma \\ &= 8.68 \times 10^8\, \rho \exp(-15.43/T_9) \text{ sec}^{-1} \text{ continuum, } 1 \leq T_9 \leq 5\end{aligned} \qquad 97.$$

$$\tau_{\alpha\gamma}(Ne^{20}) = \frac{4.61 \times 10^{-4}}{\rho X_{He^4}} \exp(15.43/T_9) \text{ sec continuum, } 1 \leq T_9 \leq 5 \qquad 98.$$

TABLE VI

CHARGED-PARTICLE CONTINUUM-REACTION DATA

Reaction		$F^{19}(p,\gamma)Ne^{20}$	$Ne^{20}(\alpha,\gamma)Mg^{24}$	$Na^{23}(p,\gamma)Mg^{24}$	$Na^{23}(p,\alpha)Ne^{20}$	$Mg^{24}(\alpha,\gamma)Si^{28}$	$Al^{27}(p,\gamma)Si^{28}$
Parameters in typical equations indicated in parentheses							
Q	(8)	12.844	9.317	11.694	2.377	9.981	11.583
$(Q/k)_9$	(100)	1.491*E* 02	1.081*E* 02	1.357*E* 02	2.759*E* 01	1.158*E* 02	1.344*E* 02
E_{th} CM	(95)	0.537	1.330	0.435	0.380	1.329	0.526
$(E_{th}/k)_9$	(97)	6.235	1.543*E* 01	5.042	4.408	1.543*E* 01	6.100
E_{th}' CM	(96)	13.382	10.647	12.129	2.757	11.311	12.108
$(E_{th}'/k)_9$	(100)	1.553*E* 02	1.236*E* 02	1.407*E* 02	3.200*E* 01	1.313*E* 02	1.405*E* 02
Coefficients in typical equations indicated in parentheses							
⟨01⟩	(97)	1.24 *E*−19	1.44 *E*−20	1.13 *E*−19	1.82 *E*−18	2.85 *E*−21	1.92 *E*−19
[01]	(97)	7.46 *E* 04	8.68 *E* 03	6.79 *E* 04	1.10 *E* 06	1.72 *E* 03	1.16 *E* 05
⟨23⟩	(101)	—	—	—	2.27 *E*−18	—	—
$[\lambda_\gamma]$	(100)(101)	2.76 *E* 15	5.22 *E* 14	5.08 *E* 15	1.37 *E* 06	1.08 *E* 14	1.31 *E* 16
$\tau_1(0)$	(98)	1.35 *E*−05	4.61 *E*−04	1.49 *E*−05	9.18 *E*−07	2.33 *E*−03	8.71 *E*−06
$\tau_0(1)$	(98)	2.55 *E*−04	2.30 *E*−03	3.39 *E*−04	2.09 *E*−05	1.40 *E*−02	2.33 *E*−04
$\mathcal{E}_{01}$	(99)	4.83 *E* 22	9.75 *E* 20	3.30 *E* 22	1.09 *E* 23	1.72 *E* 20	4.76 *E* 22
Uncertainty		±50%	FAC 2	±50%	±50%	±50%	±50%
T_9 limits		1 to 5	1 to 5	1 to 5	1 to 5	1 to 5	1 to 5
References		Aj 59	Sm 65	Pr 62, En 62 Gl 62	Ku 63, En 62	Sm 62, En 62 We 64	An 63, En 62

Reaction		$Al^{27}(p,\alpha)Mg^{24}$	$Si^{28}(\alpha,\gamma)S^{32}$	$P^{31}(p,\gamma)S^{32}$	$P^{31}(p,\alpha)Si^{28}$	$Cl^{35}(p,\gamma)Ar^{36}$	$K^{39}(p,\gamma)Ca^{40}$
Parameters in typical equations indicated in parentheses							
Q	(8)	1.601	6.948	8.864	1.917	8.506	8.333
$(Q/k)_9$	(100)	1.858*E* 01	8.063*E* 01	1.029*E* 02	2.224*E* 01	9.871*E* 01	9.671*E* 01
E_{th} CM	(95)	0.947	1.900	0.521	1.031	0.551	0.910
$(E_t/k)_9$	(97)	1.099*E* 01	2.205*E* 01	6.044	1.197*E* 01	6.392	1.056*E* 01
E_t' CM	(96)	2.549	8.848	9.385	2.948	9.057	9.243
$(E_{th}'/k)_9$	(100)	2.958*E* 01	1.027*E* 02	1.089*E* 02	3.421*E* 01	1.051*E* 02	1.073*E* 02
Coefficients in typical equations indicated in parentheses							
⟨01⟩	(97)	7.05 *E*−18	6.71 *E*−22	3.25 *E*−20	1.31 *E*−17	4.43 *E*−20	9.61 *E*−20
[01]	(97)	4.24 *E* 06	4.04 *E* 02	1.96 *E* 04	7.92 *E* 06	2.67 *E* 04	5.78 *E* 04
⟨23⟩	(101)	1.27 *E*−17	—	—	7.74 *E*−18	—	—
$[\lambda_\gamma]$	(100)(101)	7.68 *E* 06	2.61 *E* 13	7.44 *E* 14	4.66 *E* 06	2.04 *E* 15	4.44 *E* 15
$\tau_1(0)$	(98)	2.37 *E*−07	9.91 *E*−03	5.15 *E*−05	1.27 *E*−07	3.78 *E*−05	1.74 *E*−05
$\tau_0(1)$	(98)	6.36 *E*−06	6.93 *E*−02	1.58 *E*−03	3.91 *E*−06	1.31 *E*−03	6.74 *E*−04
$\mathcal{E}_{01}$	(99)	2.41 *E* 23	2.42 *E* 19	5.36 *E* 21	4.69 *E* 23	6.21 *E* 21	1.18 *E* 22
Uncertainty		±50%	FAC 3	±50%	±50%	±50%	±50%
T_9 limits		1 to 5	1 to 5	1 to 5	1 to 5	1 to 5	1 to 5
References		Ab 63, En 62	Sm 64	Sm 64, Sp 65	Ku 63, En 62	Er 65	Le 66

Units: Energy in MeV; ⟨ ⟩ in cm^3 sec^{-1}; [ ] in sec^{-1}); τ in sec; $\mathcal{E}_{01}$ in erg g^{-1} sec^{-1}; $T_9 = T/10^9$.

$$\mathcal{E}(\mathrm{Ne}^{20}\alpha)_\gamma = 9.75 \times 10^{20}\,\rho X_{\mathrm{He}^4} X_{\mathrm{Ne}^{20}} \exp(-15.43/T_9)\ \mathrm{erg\ g^{-1}\ sec^{-1}} \quad \text{continuum, } 1 \leq T_9 \leq 5 \qquad 99.$$

$$\frac{1}{\tau_{\gamma\alpha}(\mathrm{Mg}^{24})} = \lambda_{\gamma\alpha}(\mathrm{Mg}^{24}) = 5.22 \times 10^{14}\,T_9^{3/2} \exp(-15.43/T_9 - 108.1/T_9)$$

$$= 5.22 \times 10^{14}\,T_9^{3/2} \exp(-123.6/T_9)\ \mathrm{sec^{-1}}\ \text{continuum, } 1 \leq T_9 \leq 5 \qquad 100.$$

$$[\mathrm{Ne}^{20}\alpha]_p = 6.0225 \times 10^{23}\,\rho\,\langle \mathrm{Ne}^{20}\alpha\rangle_p$$

$$= 1.37 \times 10^{6}\,\rho \exp(-32.00/T_9)\ \mathrm{sec^{-1}}\ \text{continuum, } 1 \leq T_9 \leq 5 \qquad 101.$$

General Discussion

The section just concluded has presented in tabular form the currently known empirical data on the reaction rates for a number of nuclear reactions involving neutrons, protons, and alpha particles. In this section we discuss the relevance of these data to thermonuclear processes occurring under astrophysical circumstances.

The proton-proton chain.—Energy generation in the Sun and other stars with central temperature less than $\sim 2 \times 10^7$ °K occurs through the *pp* chain, the rate for which is basically determined by the cross section for the $\mathrm{H}^1(p,e^+\nu)\mathrm{D}^2$ reaction which must be calculated theoretically. However, the calculations are based on well-determined empirical parameters for proton-proton scattering and for Gamow-Teller beta decay, and we have assigned an uncertainty of only ± 10 per cent to the calculated rate.

Bahcall (1964) has estimated the rate of $\mathrm{H}^1(pe^-,\nu)\mathrm{D}^2$ to be $\sim 3.4 \times 10^{-6} \cdot \rho(1+X_\mathrm{H})/2T_9^{1/2}$ times that for $\mathrm{H}^1(p,e^+\nu)$ so that the *pep* process contributes significantly to hydrogen burning at $T_9 \sim 0.01$ only under the rather special circumstances that $\rho > 10^4$ g cm^{-3}. Werntz & Brennan (1967) have shown that another "weak" interaction process which may occur in the *pp* chain, namely $\mathrm{He}^3(p,e^+\nu)\mathrm{He}^4$, has an upper limit calculated from the rate of the analogous reaction $\mathrm{He}^3(n,\gamma)\mathrm{He}^4$ which makes it unimportant under most circumstances. Bahcall & Wolf (1964) have shown that $\mathrm{He}^3(e^-,\nu)\mathrm{T}^3$ and subsequent tritium-burning reactions are important only in the late stages of hydrogen burning in stars of small mass.

An important and competitive weak interaction is the capture of electrons by Be^7. The rate for this capture under astrophysical circumstances has been calculated from the decay rate of atomic Be^7 by Bahcall (1962) who gives an approximate expression

$$\lambda_e(\mathrm{Be}^7) = 6.70 \times 10^{-11}\,\rho(1 + X_\mathrm{H})T_9^{-1/2}\ \mathrm{sec^{-1}} \qquad T_9 \leq 1 \qquad 102.$$

which is valid under circumstances in which electrons are nondegenerate and nonrelativistic.

The rates of all "strong" reactions (Fowler & Vogl 1964) in the *pp* chain have now been measured with considerable precision. Recent measurements by Parker (1966) show that the cross section for the first reaction in the overall process, $\mathrm{Be}^7(p,\gamma)\mathrm{B}^8(e^+\nu)\mathrm{Be}^{8*}(\alpha)\mathrm{He}^4$, is somewhat greater than heretofore thought. This result is important in determining the competition of this process with $Be^7(e^-,\nu)Li^7(p,\alpha)He^4$ in the burning of Be^7. Even more recent measurements of Winkler & Dwarakanath (1967) yield a considerably larger and more precise cross-section factor, $S(0) = 5 \pm 1$ MeV-barn, for the $He^3(\tau,2p)He^4$ reaction than the value $S(0) \sim 1$ MeV given by earlier measurements of Good, Kunz & Moak (1954). This changes the competition of this reaction with $He^4(\tau,\gamma)Be^7$.

It will be clear that important modifications have been introduced in the production rates of Be^7 and B^8. The neutrinos emitted in the decay of these two nuclei in the Sun are those which will be detected at the new solar-neutrino observatory under construction by Davis (1964). A detailed re-evaluation of the neutrino flux from the Sun for comparison with the imminent observations by Davis (1964) is now under way by Shaviv, Bahcall & Fowler (1967). Calculations based on previous results have been made by Sears (1964) and Bahcall (1966) and have been discussed by Bahcall & Davis (1966).

The CNO bi-cycle.—The new results reported here do not significantly change the equilibrium abundances calculated for the CNO bi-cycle by Caughlan & Fowler (1962) although, for example, $(C^{12}/C^{13}) = 3.4$ rather than 4.0 at equilibrium at 2×10^7 °K. More importantly, the conclusion is still inescapable that the CNO bi-cycle under ordinary circumstances processes C^{12} and O^{16} primarily into N^{14} with the production of only relatively small amounts of C^{13}, N^{15}, and O^{17}. Measurements by Honsaker (1960), Clayton (1962), and Brown (1962a) exorcised a number of ghostly resonances in the bi-cycle. Recent measurements by Hensley (1967) have shown that the sixth excited state in O^{15}, found to be perilously close to the $N^{14}(p,\gamma)O^{15}$ threshold by Warburton, Olness & Alburger (1965), is actually *bound* by 21.6 ± 1.1 keV and requires d-wave ($l = 2$) protons for formation of $N^{14} + p$ in any case. Consequently the wing of the state makes a negligible contribution to the rate of $N^{14}(p,\gamma)O^{15}$, and the conclusions stand that the overall rate of the bi-cycle is basically determined by the rate of this reaction and that N^{14} is the main constituent (>92 per cent) of the CNO isotopes at equilibrium under ordinary circumstances. Caughlan (1965) has shown that the production of excess N^{14} can be avoided only by limiting C^{12} or O^{16} to an exposure of less than one proton per nucleus involved.

Helium burning and subsequent processes.—It is important to emphasize that the rate of the Salpeter-Hoyle process, $3He^4 \rightarrow C^{12}$, is basically determined by beautiful but difficult measurements on the monopole transition in the inelastic scattering of electrons by C^{12} involving the key excited state in this nucleus at 7.644 MeV (Crannell & Griffy 1964). From the monopole matrix element for $C^{12} \rightarrow C^{12*}$ the partial width for electron-positron pair emission from $C^{12*} \rightarrow C^{12}$ can be calculated by reciprocity arguments. The widths for alpha-particle and gamma-ray emission by the excited state are then empirically determined relative to the pair-emission width. The electron-positron pair emission is relatively unimportant in the decay of C^{12*}(7.644 MeV) but the "inverse" monopole transition is the only one for which an absolute determination has been made, and all other transition widths are determined relative to the monopole matrix element. This is a superb illustration of the importance of detailed nuclear spectroscopy in astrophysics. The information given on page 558 reflects the latest in empirical results but does not greatly change the rate of the $3He^4 \rightarrow C^{12}$ process from the previously accepted value.

The rate of the next step in helium burning, $C^{12}(\alpha,\gamma)O^{16}$, has long been very uncertain because of the impossibility of directly measuring the reduced alpha-particle-emission width θ_α^2 for the bound excited state in O^{16} at 7.115 MeV. B^2FH (1957) made an order-of-magnitude estimate, $\theta_\alpha^2 \sim 0.1$, primarily on the basis that the reduced width is not expected to exceed unity and should not be less than 1 per cent on any reasonable model of the O^{16} nucleus. Fowler & Hoyle (1964) showed that attempts to evaluate θ_α^2 for the 7.115-MeV state by averaging over measured widths for other states were fallacious in principle. As discussed above, a valid theoretical calculation and an indirect measurement in $Li^6(C^{12},d)O^{16}$ are consistent with $\theta_\alpha^2 = 0.085 \pm 0.04$.

The uncertainty in the value for θ_α^2 is still relatively large, but reasonable conclusions can now be drawn concerning the end result of helium burning. This problem has been investigated most recently by Deinzer & Salpeter (1964). With the new input data their results indicate approximately equal production of C^{12} and O^{16} at the end of helium burning in stars with mass up to 10 $M_\odot$. The C^{12} production is less for greater masses, dropping to 10 per cent by mass at 100 $M_\odot$. Small amounts of Ne^{20} and Mg^{24} are also produced in stars with $M > 10$ $M_\odot$ but O^{16} is the major product, 60 per cent by mass in stars with $M = 100$ $M_\odot$.

It will be clear that helium burning will be succeeded by carbon burning ($C^{12}+C^{12}$) and eventually by oxygen burning ($O^{16}+O^{16}$) in giant stars for all stellar masses of practical interest. We have abstained from tabulating the rates of these processes on the grounds that it is not presently feasible to extrapolate the experimental results obtained at relatively high bombardment energies by Bromley, Kuehner & Almqvist (1960) and by Almqvist, Bromley & Kuehner (1960) into the relevant stellar-energy range. Efforts to extend the experimental determinations to lower energies are now being made by Patterson, Winkler & Zaidins (1967). We believe that the question whether carbon burning can occur in red supergiants at a temperature low enough to avoid large neutrino-energy loss through $e^+ + e^- \rightarrow \nu + \bar{\nu}$ is still open. The problem has been discussed in detail by Hayashi, Hōshi & Sugimoto (1962).

Silicon burning.—Carbon and oxygen burning lead eventually to the production mainly of Si^{28} but with smaller amounts of other intermediate-mass nuclei. Silicon burning proceeds through a complicated chain of events involving the photoproduction and radiative capture of alpha particles, protons, and neutrons. The process was originally dubbed the alpha process by B^2FH but a better terminology is just silicon burning. Tables II through VI contain data on the key photodisintegration rates for S^{32}, Si^{28}, Mg^{24}, Ne^{20}, O^{16}, and C^{12}. The buildup from Si^{28} to the iron-group nuclei through a quasi-equilibrium process is under investigation by Bodansky, Clayton & Fowler (1967).

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We have taken every possible care in this research to avoid errors in the accumulation and presentation of the numerical data tabulated in Tables II through VI. In this connection, however, we wish to draw the reader's attention to the following quotation:

> Research, of course, is no substitute for wisdom.
>
> William Manchester (1966)

SUPPLEMENTARY TABLES

At the request of many colleagues we append supplements to Tables II through VI. These supplements give the numerical coefficients in the reaction rate equations for P_{01}, P_{23}, $P_{2\gamma}$, or P_{234} in reactions cm^{-3} sec^{-1}.

In form the equations are very similar, except for an additional factor ρ, to those throughout the main text for $\mathcal{E}$, the energy generation. For example, compare the following expression with Equation 81:

$$\begin{aligned} P(C^{12}p)_\gamma = {} & 1.02 \times 10^{30} (1 + 0.0304\, T_9^{1/3} + 1.19\, T_9^{2/3} + 0.254\, T_9 \\ & + 2.06\, T_9^{4/3} + 1.12\, T_9^{5/3}) \\ & \times \rho^2 X_H X_{C^{12}} T_9^{-2/3} \exp(-13.69/T_9^{1/3}) \qquad 0 \le T_9 \le 0.55 \\ & + 5.38 \times 10^{27} \rho^2 X_H X_{C^{12}}\, T_9^{-3/2} \exp(-4.925/T_9) \qquad 0.25 \le T_9 \le 7 \\ & \text{reactions cm}^{-3}\text{ sec}^{-1} \end{aligned} \qquad 103.$$

TABLE II SUPPLEMENT

Neutron Nonresonant-Reaction Data
Coefficients in Reaction Rates per cm³ sec

Reaction	$H^1(n,\gamma)D^2$	$D^2(n,\gamma)T^3$	$He^3(n,\gamma)He^4$	$He^3(n,p)T^3$	$Li^6(n,\gamma)Li^7$	$Li^6(n,t)He^4$
P_{01}	2.61 E 28	1.96 E 25	1.31 E 24	1.40 E 32	5.92 E 26	1.24 E 31
$P_{23}\ P_{2\gamma}\ P_{234}$	6.20 E 37	2.16 E 35	2.61 E 34	1.40 E 32	6.09 E 36	6.68 E 30
Reaction	$Li^7(n,\gamma)Li^8$	$Be^7(n,p)Li^7$	$Be^9(n,\gamma)Be^{10}$	$B^{10}(n,\gamma)B^{11}$	$B^{10}(n,\alpha)Li^7$	$B^{11}(n,\gamma)B^{12}$
P_{01}	4.17 E 26	5.75 E 32	8.34 E 25	3.95 E 27	3.03 E 31	3.59 E 25
$P_{23}\ P_{2\gamma}\ P_{234}$	4.81 E 36	5.77 E 32	5.17 E 36	1.10 E 38	8.23 E 30	7.76 E 35
Reaction	$C^{12}(n,\gamma)C^{13}$	$C^{13}(n,\gamma)C^{14}$	$N^{14}(n,\gamma)N^{15}$	$N^{14}(n,p)C^{14}$	$N^{15}(n,\gamma)N^{16}$	$O^{16}(n,\gamma)O^{17}$
P_{01}	2.24 E 25	5.48 E 24	4.24 E 26	1.02 E 28	1.27 E 23	8.80 E 23
$P_{23}\ P_{2\gamma}\ P_{234}$	1.85 E 35	1.83 E 35	1.08 E 37	3.08 E 28	8.68 E 32	2.54 E 33
Reaction	$O^{17}(n,\alpha)C^{14}$	$O^{18}(n,\gamma)O^{19}$	$F^{19}(n,\gamma)F^{20}$	$Ne^{22}(n,\alpha)O^{18}$	$Ne^{22}(n,\gamma)Ne^{23}$	$Na^{23}(n,\gamma)Na^{24}$
P_{01}	1.09 E 27	9.23 E 23	4.08 E 25	3.62 E 29	1.30 E 26	1.84 E 27
$P_{23}\ P_{2\gamma}\ P_{234}$	6.80 E 26	2.71 E 33	2.89 E 35	1.36 E 29	3.89 E 35	1.48 E 37

TABLE III SUPPLEMENT

Charged-Particle Nonresonant-Reaction Data
Coefficients in Reaction Rates per cm³ sec

Reaction	$H^1(p,e^+\nu)D^2$	$H^1(pe^-,\nu)D^2$	$He^3(p,e^+\nu)He^4$	$D^2(p,\gamma)He^3$	$D^2(d,n)He^3$	$D^2(d,p)T^3$
P_{01}	9.81 E 08	3.35 E 03	7.94 E 10	6.63 E 26	3.07 E 31	3.07 E 31
$P_{23}\ P_{2\gamma}\ P_{234}$	—	—	—	7.29 E 36	1.42 E 32	1.42 E 32
Reaction	$D^2(d,\gamma)He^4$	$T^3(p,\gamma)He^4$	$T^3(d,n)He^4$	$He^3(d,p)He^4$	$He^4(t,\gamma)Li^7$	$He^4(\tau,\gamma)Be^7$
P_{01}	1.29 E 23	4.36 E 27	8.02 E 33	6.43 E 33	2.63 E 28	2.43 E 29
$P_{23}\ P_{2\gamma}\ P_{234}$	1.18 E 34	8.64 E 37	6.68 E 34	5.37 E 34	5.04 E 38	4.66 E 39
Reaction	$T^3(t,2n)He^4$	$He^3(t,np)He^4$	$He^3(\tau,2p)He^4$	$He^3(t,d)He^4$	$Li^6(p,\tau)He^4$	$Li^7(p,\alpha)He^4$
P_{01}	3.62 E 31	3.70 E 32	1.79 E 33	2.56 E 32	6.48 E 33	1.25 E 32
$P_{23}\ P_{2\gamma}\ P_{234}$	2.73 E 22	2.81 E 23	1.36 E 24	4.62 E 32	3.48 E 33	1.30 E 32
Reaction	$C^{12}(p,\gamma)N^{13}$	$C^{13}(p,\gamma)N^{14}$	$N^{13}(p,\gamma)O^{14}$	$N^{14}(p,\gamma)O^{15}$	$N^{15}(p,\gamma)O^{16}$	$N^{15}(p,\alpha)C^{12}$
P_{01}	1.02 E 30	3.68 E 30	1.94 E 30	1.80 E 30	1.67 E 31	3.25 E 34
$P_{23}\ P_{2\gamma}\ P_{234}$	8.37 E 39	4.10 E 40	6.47 E 40	4.56 E 40	5.71 E 41	7.22 E 33
Reaction	$Be^7(p,\gamma)B^8$	$Be^9(p,d)Be^8$	$Be^9(p,\alpha)Li^6$	$B^{11}(p,2\alpha)He^4$	$N^{14}(\alpha,\gamma)F^{18}$	$O^{18}(\alpha,\gamma)Ne^{22}$
P_{01}	4.42 E 28	1.28 E 34	1.28 E 34	7.47 E 34	1.21 E 39	8.63 E 39
$P_{23}\ P_{2\gamma}\ P_{234}$	5.08 E 38	8.11 E 33	2.98 E 33	7.54 E 23	2.05 E 50	1.65 E 51
Reaction	$B^{10}(p,\alpha)Be^7$	$C^{13}(\alpha,n)O^{16}$	$O^{16}(p,\gamma)F^{17}$	$O^{16}(\alpha,\gamma)Ne^{20}$	$O^{17}(p,\alpha)N^{14}$	$Ne^{20}(p,\gamma)Na^{21}$
P_{01}	7.51 E 33	7.83 E 37	6.14 E 30	1.26 E 31	6.72 E 31	2.81 E 31
$P_{23}\ P_{2\gamma}\ P_{234}$	2.03 E 33	1.46 E 39	1.77 E 40	2.28 E 42	1.39 E 31	1.25 E 41

TABLE IV SUPPLEMENT

NEUTRON RESONANT-REACTION DATA
COEFFICIENTS IN REACTION RATES PER CM^3 SEC

Reaction	$He^{3}(n,p)T^{3}$	$Li^{6}(n,t)He^{4}$	$Li^{7}(n,\gamma)Li^{8}$	$B^{11}(n,\gamma)B^{12}$	$N^{14}(n,p)C^{14}$	$N^{14}(n,p)C^{14}$
P_{01}	2.56 *E* 34	3.63 *E* 32	8.47 *E* 26	3.44 *E* 27	3.28 *E* 30	1.59 *E* 31
$P_{22}\ P_{2\gamma}\ P_{234}$	2.56 *E* 34	1.95 *E* 32	9.78 *E* 36	7.45 *E* 37	9.87 *E* 30	4.79 *E* 31

TABLE V SUPPLEMENT

CHARGED-PARTICLE RESONANT-REACTION DATA
COEFFICIENTS IN REACTION RATES PER CM^3 SEC

Reaction	$T^{3}(d,n)He^{4}$	$He^{3}(d,p)He^{4}$	$Li^{7}(p,\gamma)Be^{8}$	$Li^{7}(\alpha,\gamma)B^{11}$	$Li^{7}(\alpha,\gamma)B^{11}$	$Li^{7}(\alpha,\gamma)B^{11}$
P_{01}	1.20 *E* 32	2.56 *E* 32	1.31 *E* 29	3.65 *E* 25	4.47 *E* 26	2.84 *E* 27
$P_{22}\ P_{2\gamma}\ P_{234}$	9.98 *E* 32	2.14 *E* 33	7.60 *E* 39	3.74 *E* 36	4.58 *E* 37	2.91 *E* 38
Reaction	$Be^{7}(p,\gamma)B^{8}$	$Be^{7}(\alpha,\gamma)C^{11}$	$Be^{7}(\alpha,\gamma)C^{11}$	$Be^{9}(p,d)Be^{8}$	$Be^{9}(p,d)Be^{8}$	$Be^{9}(p,d)Be^{8}$
P_{01}	2.78 *E* 26	1.56 *E* 27	3.90 *E* 27	7.73 *E* 31	9.74 *E* 31	3.73 *E* 32
$P_{22}\ P_{2\gamma}\ P_{234}$	3.19 *E* 36	1.60 *E* 38	3.99 *E* 38	4.91 *E* 31	6.19 *E* 31	2.37 *E* 32
Reaction	$Be^{9}(p,\alpha)Li^{6}$	$Be^{9}(p,\alpha)Li^{6}$	$Be^{9}(\alpha,n)C^{12}$	$Be^{9}(\alpha,n)C^{12}$	$Be^{9}(\alpha,n)C^{12}$	$Be^{9}(\alpha,n)C^{12}$
P_{01}	5.98 *E* 31	1.21 *E* 32	2.11 *E* 27	4.79 *E* 26	3.14 *E* 31	4.44 *E* 31
$P_{22}\ P_{2\gamma}\ P_{234}$	1.39 *E* 31	2.81 *E* 31	6.47 *E* 28	1.47 *E* 28	9.62 *E* 32	1.36 *E* 33
Reaction	$B^{10}(p,\alpha)Be^{7}$	$B^{10}(p,\alpha)Be^{7}$	$B^{11}(p,\gamma)C^{12}$	$B^{11}(p,\gamma)C^{12}$	$B^{11}(p,\gamma)C^{12}$	$B^{11}(p,2\alpha)He^{4}$
P_{01}	2.17 *E* 32	3.47 *E* 32	5.00 *E* 26	3.02 *E* 28	2.60 *E* 29	3.24 *E* 28
$P_{22}\ P_{2\gamma}\ P_{234}$	5.88 *E* 31	9.40 *E* 31	3.24 *E* 37	1.96 *E* 39	1.68 *E* 40	3.27 *E* 17
Reaction	$B^{11}(p,2\alpha)He^{4}$	$B^{11}(p,2\alpha)He^{4}$	$C^{12}(p,\gamma)N^{13}$	$C^{13}(p,\gamma)N^{14}$	$C^{13}(p,\gamma)N^{14}$	$Ne^{20}(p,\gamma)Na^{21}$
P_{01}	3.63 *E* 32	7.65 *E* 32	5.38 *E* 27	6.21 *E* 28	1.04 *E* 29	4.90 *E* 27
$P_{22}\ P_{2\gamma}\ P_{234}$	3.66 *E* 21	7.72 *E* 21	4.43 *E* 37	6.92 *E* 38	1.16 *E* 39	2.18 *E* 37
Reaction	$N^{14}(p,\gamma)O^{15}$	$N^{14}(p,\gamma)O^{15}$	$N^{14}(p,\gamma)O^{15}$	$C^{13}(\alpha,n)O^{16}$	$C^{13}(\alpha,n)O^{16}$	$C^{13}(\alpha,n)O^{16}$
P_{01}	9.74 *E* 25	2.66 *E* 27	2.17 *E* 29	2.45 *E* 28	2.22 *E* 29	5.50 *E* 28
$P_{22}\ P_{2\gamma}\ P_{234}$	2.47 *E* 36	6.76 *E* 37	5.52 *E* 39	4.58 *E* 29	4.15 *E* 30	1.03 *E* 30
Reaction	$N^{15}(p,\gamma)O^{16}$	$N^{15}(p,\gamma)O^{16}$	$N^{15}(p,\alpha)C^{12}$	$N^{15}(p,\alpha)C^{12}$	$N^{15}(p,\alpha)C^{12}$	$O^{17}(p,\alpha)N^{14}$
P_{01}	4.64 *E* 26	2.93 *E* 29	5.14 *E* 30	1.37 *E* 32	3.95 *E* 31	2.76 *E* 20
$P_{22}\ P_{2\gamma}\ P_{234}$	1.59 *E* 37	1.00 *E* 40	1.14 *E* 30	3.04 *E* 31	8.77 *E* 30	5.71 *E* 19
Reaction	$C^{12}(\alpha,\gamma)O^{16}$	$C^{12}(\alpha,\gamma)O^{16}$	$C^{12}(\alpha,\gamma)O^{16}$	$O^{16}(\alpha,\gamma)Ne^{20}$	$O^{16}(\alpha,\gamma)Ne^{20}$	$O^{16}(\alpha,\gamma)Ne^{20}$
P_{01}	2.45 *E* 25	1.32 *E* 25	1.54 *E* 26	2.83 *E* 23	2.43 *E* 24	6.68 *E* 24
$P_{22}\ P_{2\gamma}\ P_{234}$	3.78 *E* 36	2.03 *E* 36	2.37 *E* 37	5.13 *E* 34	4.39 *E* 35	1.21 *E* 36

TABLE VI SUPPLEMENT

CONTINUUM-REACTION DATA
COEFFICIENTS IN REACTION RATES PER CM³ SEC

Reaction	$F^{19}(p,\gamma)Ne^{20}$	$Ne^{20}(\alpha,\gamma)Mg^{24}$	$Na^{23}(p,\gamma)Mg^{24}$	$Na^{23}(p,\alpha)Ne^{20}$	$Mg^{24}(\alpha,\gamma)Si^{28}$	$Al^{27}(p,\gamma)Si^{28}$
P_{01}	2.35 *E* 27	6.54 *E* 25	1.76 *E* 27	2.85 *E* 28	1.08 *E* 25	2.56 *E* 27
$P_{21}\,P_{3\gamma}\,P_{2\Delta}$	8.31 *E* 37	1.31 *E* 37	1.28 *E* 38	1.03 *E* 28	2.32 *E* 36	2.82 *E* 38
Reaction	$Al^{27}(p,\alpha)Mg^{24}$	$Si^{28}(\alpha,\gamma)S^{32}$	$P^{31}(p,\gamma)S^{32}$	$P^{31}(p,\alpha)Si^{28}$	$Cl^{35}(p,\gamma)Ar^{36}$	$K^{39}(p,\gamma)Ca^{40}$
P_{01}	9.40 *E* 28	2.17 *E* 24	3.77 *E* 26	1.53 *E* 29	4.56 *E* 26	8.87 *E* 26
$P_{21}\,P_{3\gamma}\,P_{2\Delta}$	4.81 *E* 28	4.92 *E* 35	1.40 *E* 37	2.51 *E* 28	3.42 *E* 37	6.70 *E* 37

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