6 Light from the Stars

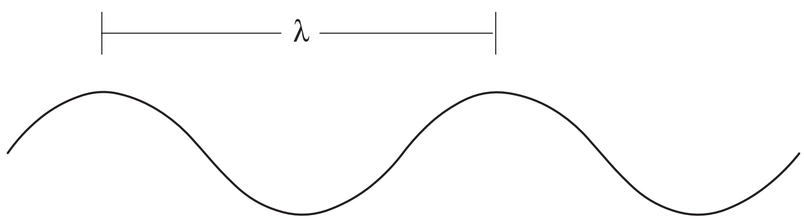
Essentially everything that we know about objects in the sky is because of the light coming from them.

6.1 The Electromagnetic Spectrum

The properties of light (electromagnetic waves) are central to our understanding of astronomy.

6.1.1 Light as a wave

Light may be viewed as either a stream of massless particles (*photons*) or a wave in the electromagnetic field.



The wavelength of light is denoted by λ. The SI unit of length is the meter, but it is common to use for wavelengths the units

$$1 \text{ nanometer (nm)} = 10^{-9} \text{ m} \qquad 1 \text{ Angstrom (Å)} = 10^{-10} \text{ m}$$

(Thus, 1 nm = 10 Å.)

6.1.2 Frequency of light waves

In terms of the wavelength, the frequency ν of a light wave (frequency with which wave-crests pass a fixed point) is given by

$$\nu = \frac{c}{\lambda}, \tag{21}$$

where c is the (constant) speed of light:

$$c = 3.0 \times 10^8 \text{ m s}^{-1} \qquad \text{(speed of light)}$$

The standard unit of frequency is the Hertz (Hz):

$$1 \text{ Hz} \equiv 1 \text{ cycle per second.}$$

6.1.3 Energy of light waves

The energy E of the light wave is related to its frequency and wavelength by

$$E = h\nu = \frac{hc}{\lambda}, \tag{22}$$

where h is Planck's constant:

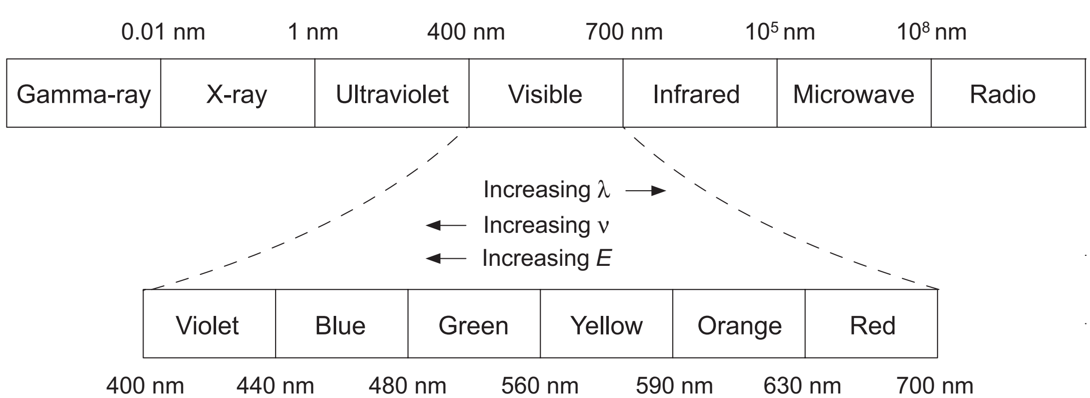
$$h = 6.625 \times 10^{-34} \text{ J s} \qquad \text{(Planck's constant)}$$

and the Joule (J) is the standard unit of energy:

$$1 \text{ Joule} \equiv 1 \text{ N m} = 1 \text{ kg m}^2\text{s}^{-2}.$$

6.1.4 Regions of the electromagnetic spectrum

Light may be classified according to its wavelength (or frequency or energy) into different regions of the *electromagnetic spectrum*. The standard classification is illustrated in the following figure (not to scale).



Example: Wavelength, frequency and energy

Consider red light with a wavelength $\lambda = 650$ nm. Its frequency is

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m s}^{-1}}{650 \times 10^{-9} \text{ m}} = 4.62 \times 10^{14} \text{ s}^{-1}$$

and its energy is

$$E = \frac{hc}{\lambda} = \frac{(6.625 \times 10^{-34} \text{ J s})(3 \times 10^8 \text{ m s}^{-1})}{650 \times 10^{-9} \text{ m}} = 3.06 \times 10^{-19} \text{ J}.$$

6.2 The Magnitude Scale

One of the fundamental quantities of interest for objects in the sky is their brightness. This is often quantified in astronomy in terms of a logarithmic scale called the *magnitude scale*.

The difference between magnitudes m_1 and m_2 for two objects is defined by

$$m_2 - m_1 = 2.5(\log b_1 - \log b_2) = 2.5 \log \frac{b_1}{b_2}, \tag{23}$$

where b_1 and b_2 are their apparent brightnesses. This is equivalent to

$$\frac{b_1}{b_2} = 10^{0.4(m_2 - m_1)}. \tag{24}$$

The normalization in the preceding equations is chosen to make the modern magnitude scale correspond at least approximately with ancient subjective scales for the brighest stars.

The following table shows the relationship of some magnitude differences $\delta m = m_2 - m_1$ and the corresponding brightness ratios.

Δm	b_1/b_2
0	1
1	2.5
2.5	10
10	10^4
25	10^{10}

6.2.1 Apparent magnitude

The following table gives the apparent visual magnitude for some objects (where visual means that the magnitude is derived form the apparent brightness in the visual part of the spectrum).

Object	m_V
Sirius (brightest star)	-1.5
Venus at brightest	-4.4
Full Moon	-12.6
The Sun	-26.8
Faintest naked-eye stars	6–7
Faintest object visible from Earth with largest conventional telescopes	~ 28
Faintest object visible from the Hubble Space Telescope	~ 30

6.2.2 Absolute magnitude

The apparent magnitude mixes up intrinsic brightness and distance effects. From basic optics, the intensity observed for a source depends on the inverse square of the distance,

$$I \propto \frac{1}{d^2}.$$

Thus, if we double the distance to an object, its apparent brightness goes down by a factor of 4.

If we know the distance d to an object we can use this rule to compute its apparent brightness and therefore its apparent magnitude, if it were placed at some other distance.

Absolute Magnitude

The absolute magnitude of an object is the apparent magnitude that it would have if it were placed at a distance of 10 pc (32.6 ly) from the observer.

This definition implies that the apparent magnitude m and absolute magnitude M of an object are related by

$$M = m - 5\log\left(\frac{d}{10\text{ pc}}\right), \tag{25}$$

or equivalently,

$$d = 10^{(m-M+5)/5}\text{ pc}, \tag{26}$$

where

$$m - M \equiv \text{ distance modulus.}$$

Some absolute magnitudes are displayed in the following table.

Object	M
Sun	$+4.8$
Betelgeuse	-5.14
Sirius	$+1.47$

6.2.3 Luminosity

The luminosity L and the absolute magnitude M are related by

$$M_2 - M_1 = 2.5\log\left(\frac{L_1}{L_2}\right), \tag{27}$$

or equivalently

$$\frac{L_1}{L_2} = 10^{0.4(M_2-M_1)}. \tag{28}$$

Luminosity has units of energy per unit time. In the SI system, the luminosity is expressed in Watts (W), with

$$1 \text{ Watt (W)} = 1 \text{ J s}^{-1}.$$

Choosing star 2 to be the Sun with $M_2 = M_{\text{sun}} = 4.85$, we may express luminosities in terms of solar luminosities through

$$\frac{L}{L_\odot} = 10^{0.4(4.85-M)}. \tag{29}$$

Example: Distance and luminosity of Rigel

For Rigel (β-Ori)

$$m_\text{V} = 0.18 \qquad M_\text{V} = -6.69$$

Therefore, the distance is

$$\begin{aligned} d &= 10^{(m-M+5)/5} = 10^{(0.18-(-6.69)+5)/5} \\ &= 10^{2.374} \\ &= 237 \text{ pc.} \end{aligned}$$

The luminosity of Rigel in solar units is

$$\begin{aligned} \frac{L}{L_\odot} &= 10^{0.4(4.85-M)} = 10^{0.4(4.85-(-6.69))} \\ &= 10^{4.616} \\ &= 41,300. \end{aligned}$$

Example: Absolute magnitude of Betelgeuse

For Betelgeuse (α-Ori)

$$m_\text{V} = 0.45 \qquad d = 131 \text{ pc.}$$

Therefore, the absolute magnitude of Betelgeuse is

$$\begin{aligned} M &= m - 5\log\left(\frac{d}{10 \text{ pc}}\right) \\ &= 0.45 - 5\log\left(\frac{131}{10}\right) \\ &= -5.14. \end{aligned}$$

The 5 stars known to have the highest luminosities are listed in the following table

Bayer name	Common name	m_V	Distance (pc)	M_V	Luminosity (Solar units)
β-Ori	Rigel	0.18	237 ± 45	-6.69	41,000
γ-Cyg	-	2.23	467 ± 112	-6.12	24,000
ζ-Pup	-	2.21	429 ± 94	-5.95	21,000
υ-Car	-	2.92	498 ± 100	-5.56	15,000
α-Car	Canopus	-0.62	95.9 ± 4.8	-5.53	14,000

Notice that to determine the luminosity we must know the distance, so only the luminosities for relatively nearby stars are known with some certainty. Even for them, the errors are often in the vicinity of 20% (see entries in the table).

6.3 Blackbody Radiation

A blackbody radiator is in perfect equilibrium with its surroundings. Stars are not exactly blackbodies, but they are approximately so.

6.3.1 The Planck law

If a body may be approximated as a blackbody, the *Planck Law* applies to the intensity of emitted radiation:

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}, \tag{30}$$

where $B_\lambda(T)$ is termed the Planck function, the wavelength is λ, the temperature is T, and

$$h = 6.625 \times 10^{-34}\ \mathrm{J\,s} \qquad \text{(Planck's constant)}$$
$$c = 3.0 \times 10^{8}\ \mathrm{m\,s^{-1}} \qquad \text{(speed of light)}$$
$$k = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}} \qquad \text{(Boltzmann's constant).}$$

> The Planck function $B_\lambda(T)$ measures the intensity of radiation emitted at a given wavelength λ in a particular direction.

Absolute Temperature Scale

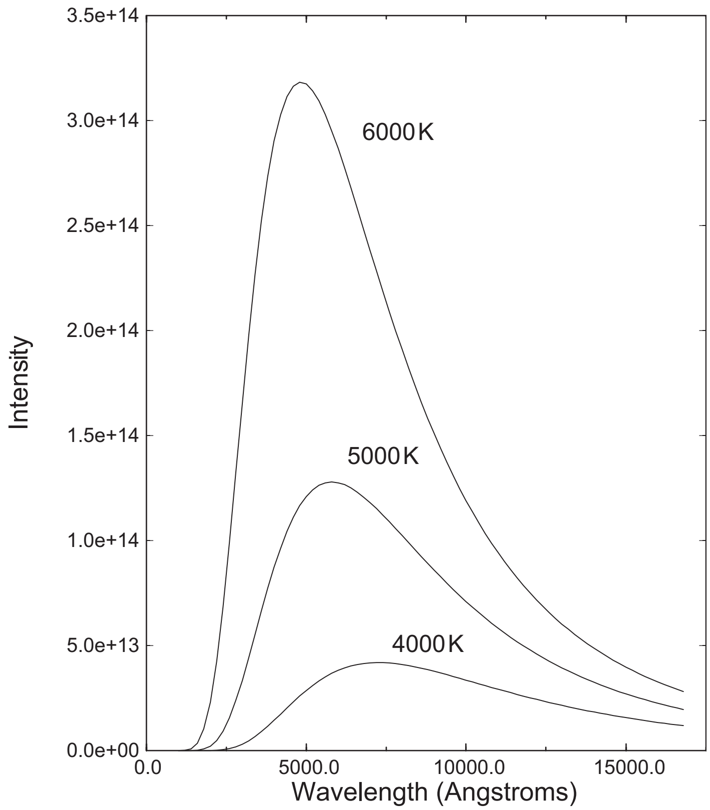
It is often convenient to measure temperatures in the *absolute* or *Kelvin* temperature scale. The Kelvin scale and the Celsius scale differ by a shift of exactly 273.15 degrees C. The units of the Kelvin scale are called "kelvins", and are abbreviated by the letter K. The relationship between a temperature in the Kelvin scale and one in the Celsius scale is

$$\mathrm{K} = {}^\circ\mathrm{C} + 273.15 \simeq {}^\circ\mathrm{C} + 273.$$

For example, the boiling point of water is 100 °C, which is 373 K

> For very large temperatures the shift of 273 degrees is negligible and the temperatures expressed as kelvins or as °C are numerically almost the same.

The blackbody spectrum associated with several temperatures is illustrated in the following figure.



Generally we see that the total area under the curve grows rapidly with temperature, and that the distribution has a single peak that shifts to shorter wavelengths as the temperature increases.

Show: Vlab 2 (Radiation Laws): Planck Law Plotter

6.3.2 The Wien law

The *Wien Displacement Law* states that for a blackbody radiator the maximum in the radiation distribution as a function of wavelength occurs at

$$\lambda_{\text{max}} = \frac{2.897 \times 10^{-3}\ \text{m K}}{T} \tag{31}$$

if T is expressed in kelvins and the wavelength in meters.

Example: Peak spectral intensity for the Sun

The surface temperature of the Sun is $T_\odot \simeq 5800$ K. Then from the Wien law we expect the peak intensity of light emitted by the Sun to occur at

$$\begin{aligned}\lambda_{\text{max}} &= \frac{2.9\times 10^{-3}\ \text{m K}}{T} = \frac{2.9\times 10^{-3}\ \text{m K}}{5800\ \text{K}}\\ &= 5\times 10^{-7}\ \text{m}\\ &= 500\ \text{nm}.\end{aligned}$$

This is in the yellow–green part of the visible spectrum, which is why the Sun looks yellow.

6.3.3 The Stefan–Boltzmann law

The *Stefan–Boltzmann Law* says that the total energy radiated per unit time per unit surface area at all wavelengths varies as the fourth power of the temperature,

$$E = \sigma T^4, \tag{32}$$

where

$$\sigma = 5.6705\times 10^{-8}\ \text{W m}^{-2}\text{K}^{-4} \quad \text{(Stefan–Boltzmann constant).} \tag{33}$$

(Recall that 1 Watt (W) = 1 J s^{-1} is a standard unit of power or luminosity.)

The total luminosity L for a spherical blackbody of radius R is then

$$L = \text{ surface area} \times \sigma T^4 = \left(4\pi R^2\right)\sigma T^4 \tag{34}$$

This equation can be solved to determine an effective surface temperature for a star, assuming it to be a spherical blackbody of radius R:

$$T_{\text{eff}} = \left(\frac{L}{4\pi R^2 \sigma}\right)^{1/4}. \tag{35}$$

Example: Effective surface temperature of the Sun

For the Sun,

$$L_\odot = 3.8\times 10^{26}\ \text{W} \qquad R_\odot = 7\times 10^{8}\ \text{m}$$

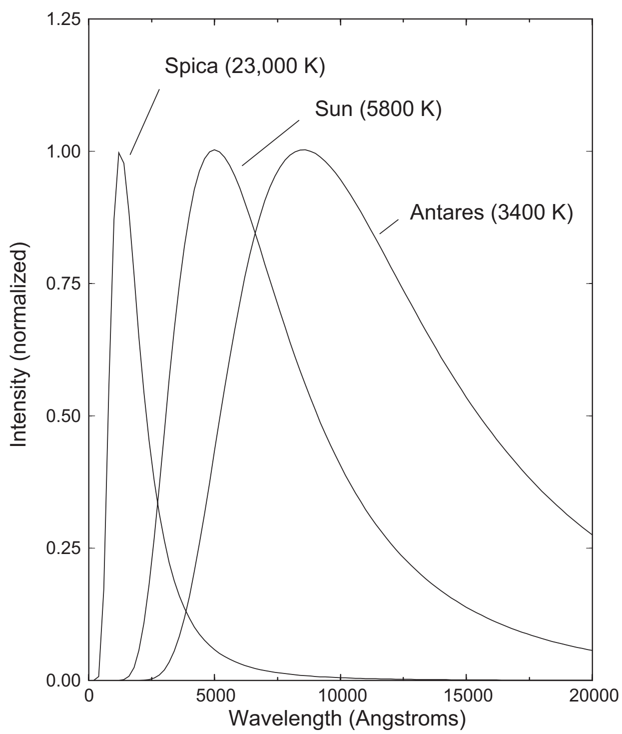
Assuming it to be a spherical blackbody, the effective surface temperature is

$$\begin{aligned} T_{\text{eff}} &= \left(\frac{L}{4\pi R^2 \sigma}\right)^{1/4} = \left(\frac{3.8 \times 10^{26}\ \text{W}}{4\pi (7 \times 10^{8}\ \text{m})^2 (5.6705 \times 10^{-8}\ \text{W}\,\text{m}^{-2}\text{K}^{-4})}\right)^{1/4} \\ &= 5789\,\text{K}. \end{aligned}$$

(The accepted value is 5780 K.)

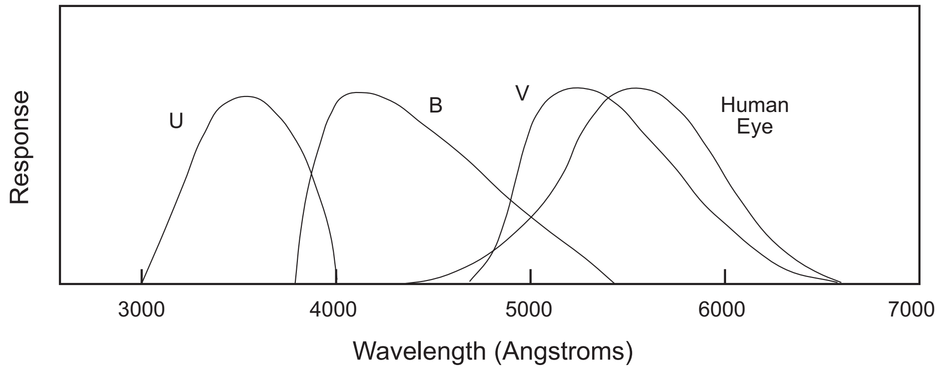
6.3.4 Color filters

From the Planck or Wein laws, we see that stars of different temperature will emit light of different relative intensities at different wavelengths. The following figure illustrates for three bright stars.



(Note that the peaks are all normalized to the same height for comparison.)

Astronomers often use *filters* on telescopes that pass only a narrow range of wavelengths. The following figure illustrates.



Note that the response of the human eye is similar to that of the V filter (and peaks in the yellow–green part of the spectrum, which is the dominant wavelength radiated by the Sun). Although not shown, the response of normal photographic film is similar to that of the B filter.

6.3.5 Differences in magnitude and the color index

The relative intensity at different wavelengths as passed by these filters clearly depends on the surface temperature of the star. This has led to the definition of quantities called color indexes for stars.

Color Index

A color index is generally the difference in magnitudes for two different ranges of wavelengths. A color index can be obtained directly at the telescope by using filters to select two different wavelength ranges and comparing the corresponding intensity.

For example, two commonly used color indexes are

$$B-V \equiv m_{\mathrm{B}} - m_{\mathrm{V}} \qquad U-B \equiv m_{\mathrm{U}} - m_{\mathrm{B}},$$

Where m_{V}, m_{B}, and m_{U} denote the apparent magnitudes determined using the V, B, and U filters, respectively.

Example: Calculation of the $B - V$ color index

For the hot blue–white star Spica:

$$B = 0.77 \qquad V = 1.0,$$

while for the cool red star Antares

$$B = 2.73 \qquad V = 0.9.$$

Then the $B - V$ color index for Spica is

$$(B - V)_{\text{Spica}} = 0.77 - 1.0 = -0.23$$

and the $B - V$ color index for Antares is

$$(B - V)_{\text{Antares}} = 2.73 - 0.9 = +1.83$$

The size of the color index is a direct measure of the surface temperature of the star. Generally, the $B - V$ color index is

- negative for the hotter stars
- positive for the cooler stars

because of the slope of the intensity curve in the visible region of the spectrum: