## 5 The Motion of Stars

In addition to the daily (diurnal) motion of the sky as a whole, the stars move (very slowly) on the celestial sphere with respect to each other. This motion is called proper motion, and is commonly denoted by the symbol $\mu$.

The motion of stars can be decomposed into a radial component (along the line of sight) and a transverse component that changes its position on the celestial sphere. It is the transverse component that produces the proper motion: the radial component of motion changes the distance of the star from us, but does change its location on the celestial sphere.

### 5.1 Proper Motion

The proper motion itself is composed of two components: a change in the right ascension and a change in the declination of the star. The following figure illustrates.


Celestial sphere ascension


The 5 stars with the largest proper motion are listed in the following table

| Star | Change in R. A. <br> $(\operatorname{arcsec} / \mathrm{yr})$ | Change in Dec <br> $(\operatorname{arcsec} / \mathrm{yr})$ | Proper Motion <br> $(\operatorname{arcsec} / \mathrm{yr})$ |
| :--- | :---: | :---: | ---: |
| Barnard's Star | -0.79784 | 10.32693 | 10.3577 |
| Kapteyn's Star | 6.50605 | -5.73139 | 8.6705 |
| Groombridge 1830 | 4.00369 | -5.81300 | 7.0584 |
| Lacaille 9352 | 6.76726 | 1.32666 | 6.8961 |
| CD-37d 15492 | 5.63407 | -2.33794 | 6.0999 |

## Example: Proper motion of Barnard's Star

Barnard's Star has the largest proper motion of any star. How long will it take for Barnard's star to change its position relative to other stars on the celestial sphere by an angle equal to the diameter of the Moon? The diameter of the Moon is about $\frac{1}{2}^{\circ}=1800$ arcsec. It will take Barnard's Star

$$
\frac{1800 \operatorname{arcsec}}{10.3 \operatorname{arcsec} \mathrm{yr}^{-1}} \simeq 175 \mathrm{yr}
$$

to move the diameter of the Moon. Most other stars move much more slowly than this on the celestial sphere.

Show OJTA animation 2.7 for proper motion of Barnard's Star.

### 5.2 Space Velocities

The true velocity of a star is called its space velocity. The space velocity $\boldsymbol{v}_{s}$ may be resolved into a radial velocity $\boldsymbol{v}_{\mathrm{r}}$ along the line of sight and a transverse velocity $\boldsymbol{v}_{\mathrm{t}}$ perpendicular to the line of sight. The space velocity and its components are illustrated in the following figure.


Their lengths are related by the Pythagorean theorem.

## Example: Space velocity for Arcturus

Typical values for the space velocities of stars are $20-100 \mathrm{~km} \mathrm{~s}^{-1}$. The star with largest known space velocity is Arcturus, in the constellation Boötes. I has a transverse velocity (determined from its change in position over time and its distance as inferred from parallax) of $119 \mathrm{~km} \mathrm{~s}^{-1}$, and a radial velocity (determined from Doppler shifts of its spectral lines) of $-5.2 \mathrm{~km} \mathrm{~s}^{-1}$ (the negative sign signifying the the radial motion is toward us). Applying the Pythagorean theorem, the magnitude of the space velocity is

$$
v_{\mathrm{s}}=\sqrt{\left(v_{\mathrm{r}}\right)^{2}+\left(v_{\mathrm{t}}\right)^{2}}=\sqrt{(-5.2)^{2}+(119)^{2}} \mathrm{~km} \mathrm{~s}^{-1}=119.1 \mathrm{~km} \mathrm{~s}^{-1} .
$$

Since the transverse velocity is much larger than the radial velocity, the direction of the space velocity is clearly almost perpendicular to our line of sight. More precisely, from trigonometry on the right triangle in the above figure, the angle between the radial direction and the space velocity vector is

$$
\theta=\cos ^{-1} \frac{\left|v_{\mathrm{r}}\right|}{\left|v_{\mathrm{s}}\right|}=\cos ^{-1}\left(\frac{5.2}{119.1}\right)=87.5^{\circ},
$$

so indeed the space velocity is almost transverse to our line of sight for Arcturus.

## When Gliese 710 Comes Calling

One fruit of the precise astrometry provided by the Hipparcos satellite was improved understanding of the space velocities for nearby stars. Presently the nearest star is 4.2 light years away. Hipparcos data indicate that at least eight nearby stars will pass closer than five light years from Earth in the next million years. One of these is Gliese 710 , now 63 light years away in the constellation Ophiuchus. It will pass within a mere light year of the Sun in about a million years. Gliese 710 is a very small, intrinsically faint star called a red dwarf. It is now much too faint to be seen without a telescope, but in a million years it will appear to be one of the brightest stars in Earth's sky because it will be so close.
The Oort comet cloud may extend out to a light year from the Sun. Thus the gravitational influence of Gliese 710, and some of the other eight stars expected to come within five light years of Earth, could disturb the Oort cloud and trigger a rain of comets into the inner Solar System.

### 5.3 Motion of the Sun Relative to the Stars

The Sun is in motion, just like other stars. It partakes of the general rotation of the galaxy (the spiral Milky Way). The Milky Way rotates once in about 225 million years, which corresponds to an average velocity for the Sun of about $220 \mathrm{~km} \mathrm{~s}^{-1}$ (the Sun covers a distance of about 4 AU in one year because of this motion). The space velocities that we measure for stars then correspond to deviations from this average motion for stars near the Sun.

## The Solar Apex and Antapex

With respect to the local field of stars, the Sun is moving at $19.7 \mathrm{~km} \mathrm{~s}^{-1}$ toward a point in the constellation Hercules (near the star Vega, which is in Lyra) that is called the solar apex. It will take us a while to get to Vega though; it is 26.5 ly away! The point on the opposite side of the sky away from which the Sun appears to be moving is called the solar antapex.

