4 Distances in Astronomy

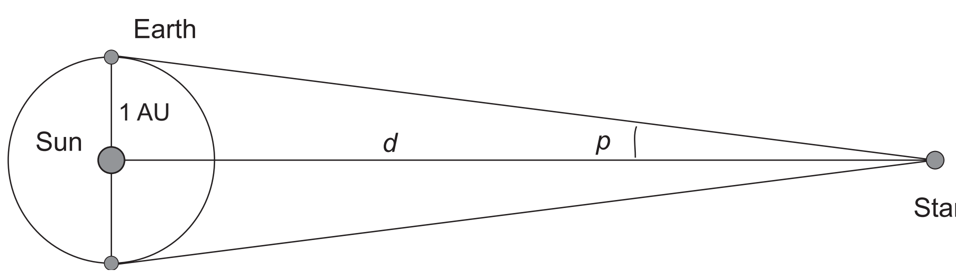
Assign: Read Chapter 3 and 5 of Carrol and Ostlie (2006)

Most of what we know about stars is derived from the light that we detect from them. How we perceive that light depends critically on the distance to stars.

Determining the distance to objects in astronomy is one of the most important tasks and yet it is one of the most difficult ones.

4.1 The Parallax Method

The most reliable way to determine distances is *trigonometric parallax*, which is illustrated in the following figure



(Not to scale!) The *parallax angle* p corresponds to a small shift on the celestial sphere because of differing vantage points as the Earth moves around its orbit. It is related to the distance d of the star through

$$\tan p = \frac{1 \text{ AU}}{d}, \tag{18}$$

where the *astronomical unit* AU is a distance unit equal to the average separation of the Earth and the Sun (the length of the Earth's semimajor orbital axis).

The first star for which parallax was measured (in 1838) is 61 Cyg (61st star from the west edge of the constellation Cygnus). It has $p = 0.286$ arcsec. This is roughly the size of your thumbnail at a distance of 15 km! That is one of the *largest* parallax shifts for stars. Generally, all stars have parallax shifts less than one arcsec. Recall that one arcsecond is about the angle subtended by a dime at a distance of 2 km.

Since the stars are very distant a small-angle approximation is well justified: for small p,

$$\sin p \simeq p \qquad \cos p \simeq 1 \quad (p \text{ in radians}).$$

Therefore, for small angles,

$$\tan p = \frac{\sin p}{\cos p} \simeq \frac{p}{1} \simeq p \quad (p \text{ in radians}).$$

Applying this small-angle approximation to the parallax formula,

$$p(\text{radians}) = \frac{1 \text{ AU}}{d}$$

4.2 Units for Distances in Astronomy

Because many distances in astronomy are very large, it is convenient to define special units that correspond to large distances.

4.2.1 Astronomical units

The astronomical unit is a common distance unit in discussing objects in the Solar System. As we have seen, one *astronomical unit* (AU) is a distance unit equal to the average separation of the Earth and the Sun (the length of the Earth's semimajor orbital axis)

$$1 \text{ AU} \sim 150,000,000 \text{ km}.$$

4.2.2 Light years

For larger distances, the *light year* (ly) is a common unit. A light year is the distance light travels in one year

$$1 \text{ light year (ly)} = \underbrace{(3.156 \times 10^7 \text{ s})}_{\text{tropical year}} \times \underbrace{(3 \times 10^5 \text{ km s}^{-1})}_{\text{speed of light}} = 9.46 \times 10^{12} \text{ km.}$$

A light year is a *distance*, not a time! The following table gives some distances expressed in light years.

Object	Distance (ly)
Nearest star (Proxima Centauri)	4.2
Diameter of our galaxy	100,000
Distance to Virgo Cluster	50,000,000

4.2.3 Parsecs

Starting from the parallax formula,

$$p(\text{radians}) = \frac{1 \text{ AU}}{d}$$

and converting the angular measure to seconds of arc

$$1 \text{ degree} = 3600 \text{ arcsec} \qquad 1 \text{ radian} = \frac{180}{\pi} \text{ degree} = 2.06265 \times 10^5 \text{ arcsec}$$

we may write

$$\frac{d}{1 \text{ AU}} = \frac{2.06265 \times 10^5}{p \text{ (arcsec)}}, \tag{19}$$

where the notation indicates that p is to be expressed in seconds of arc.

This suggests that we define a natural distance unit equal to the distance at which a star would have a parallax angle of $1''$. This unit is termed the *parsec* (from concatenating “parallax” and “seconds”), and is abbreviated by the symbol pc.

$$1 \text{ parsec (pc)} = 206,265 \text{ AU} = 3.08 \times 10^{16} \text{ m} = 3.26 \text{ ly.}$$

With these units, the distance in parsecs is just the inverse of the parallax angle in seconds of arc:

$$d \text{ (pc)} = \frac{1}{p \text{ (arc sec)}}. \tag{20}$$

Example: Distance to some nearby stars

The star Sirius is the brightest star in the sky except for the Sun. Its parallax shift is measured to be $p = 0.38$ arcseconds. Thus, its distance from Earth is

$$d = \frac{1}{p} = \frac{1}{0.38} = 2.6 \text{ pc} = 8.6 \text{ ly}.$$

α-Centauri has a parallax shift of $0.760''$. Its distance is

$$d = \frac{1}{0.76} = 1.315 \text{ pc} = 4.3 \text{ ly}.$$

Some characteristic distances expressed in parsecs are summarized in the following table.

Quantity	Distance
Average separation between stars in a galaxy	1 pc
Diameter of a large spiral galaxy	100 kpc
Separation between galaxies in a cluster of galaxies	Several Mpc
Separation between clusters of galaxies in a supercluster	10 Mpc
Distance to most distant galaxies observed	Thousands of Mpc

The distances to the 5 nearest stars are listed in the following table. Note that they are comparable to a parsec or two.

Name	Distance (ly)	Distance (pc)	Luminosity (Solar Units)
Proxima Centauri	4.2	1.3	5.3×10^{-6}
Alpha Centauri A	4.3	1.3	1.5
Alpha Centauri B	4.4	1.3	0.45
Barnard's Star	6.0	1.8	4.5×10^{-4}
Wolf 359	7.8	2.4	2.1×10^{-5}

Notice also the luminosity (intrinsic brightness) in the right column. Many of the stars near the Sun are faint.

> Notice from the preceding table how *empty* the Universe is! Even though our galaxy contains about 200 billion stars (2×10^{11}), the nearest ones to us would require 5–10 years of travel at speeds near that of light in order to reach them.

It is also common to define large multiples of the parsec,

$$1 \text{ kiloparsec (kpc)} = 10^3 \text{ pc} \qquad 1 \text{ megaparsec (Mpc)} = 10^6 \text{ pc}.$$

for very large distances. The following table summarizes some common astronomy distance units and gives their size in meters.

Quantity	Abbreviation	Distance (km)
Astronomical unit	AU	1.50×10^8
Light year	ly	9.46×10^{12}
Parsec	pc	3.08×10^{13}
Kiloparsec	kpc	3.08×10^{16}
Megaparsec	Mpc	3.08×10^{19}

Parallax Limitations

The parallax method is limited to determining distances for relatively nearby stars, because otherwise the parallax angle becomes too small to measure reliably. The best ground-based telescopes can achieve a resolution of about $0.5''$, which can sometimes be reduced to about $0.01''$ by averaging over many measurements. This corresponds to a distance of about 300 ly. Space-based telescopes can do better (see the discussion of Hipparcos below), but so far the most distant stars for which parallax has been measured reliably are less than a few thousand light years away. To determine larger distances, we must use other methods that will be discussed later.

The Hipparcos Satellite

In 1989–1993 the Hipparcos satellite measured the parallax for 120,000 stars with precision as good as 0.001 arcsec, and for another million stars with lower precision. An angle of 0.001 arcsec is comparable to the angular diameter of a golf ball viewed across the Atlantic Ocean, and corresponds to a distance of $1/0.001 \sim 1000$ pc! To set this in perspective, we knew the parallax of fewer than 1000 stars before the Hipparcos mission. Hipparcos data allow the parallax of the nearest stars to be determine with an uncertainty of only about 0.2–0.3%. Future space-based mission are expected to be more precise and to determine the parallax for even more distant stars.

The following table lists typical parallax precisions for existing and projected technology, with the corresponding maximum distance for which parallax can be used reliably.

Method	Parallax precision	Max distance
Best Earth-based	$0.01''$	100 pc
Hipparcos	$0.001''$	1 kpc
Gaia*	$0.00001''$	100 kpc
Space Interferometry Mission*	$0.000004''$	250 kpc

*Projected next-decade mission

For reference, the diameter of the visible Milky Way is about 30 kpc and the nearest galaxies are within a few hundred kpc.