1 Introduction

The first inkling of the properties that we now attribute to electricity and magnetism traces to the distant past when humans began to realize and remark upon observed physical phenomena such as the behavior of naturally occurring electricity in amber and the properties of naturally occurring magnetism in lodestones. Although these phenomena were known qualitatively to the ancient Greeks, they remained mysterious for centuries and the modern quantitative understanding of electricity and magnetism emerged over a period of only a little over a century, beginning in the late 1700s.

1.1 The Synthesis of Classical Electromagnetism

At the risk of slighting the contributions of many, one could say that the explosion in knowledge of electricity and magnetism beginning in the late 18th century, and the forging of that knowledge over the next century or so into a theoretically and mathematically coherent framework that we now call electromagnetism, can be illustrated by citing a few landmark achievements.

1. *Henry Cavendish* (1731-1810) did pioneering experiments on electrostatics in the early 1770s, including establishing that electrical forces varied as one over the square of the distance. However, his work was not widely published until after Coulomb's publication in 1785.
2. *Charles-Augustin de Coulomb* (1736-1806) published his extensive work on electrostatics beginning in 1785, including the eponymous inverse-square law for electrical interaction between charges.
3. *Hans Christian Ørsted* (1777-1827) discovered the magnetic effect of the electric current, establishing the first connection between electric and magnetic phenomena.
4. *André–Marie Ampère* (1775-1827) extended the work of Ørsted, finding in 1823 the circuit law between electric current passing through a loop and magnetic field around the loop, and establishing the formula describing the interaction of two currents.
5. *Michael Faraday* (1791-1867) did his highly influential work studying time-varying currents and fields in the mid-1800s. He discovered electromagnetic induction in 1831, which proved to be a crucial step in Maxwell's unification of electric and magnetic phenomena into modern electromagnetism.
6. *James Clerk Maxwell* (1831-1879) published his famous 1865 paper (read to the Royal Society in 1864), which unified electricity and magnetism and synthesized in equations a dynamical theory of what could now be termed the *electromagnetic field* [39, 42].

7. *Oliver Heaviside* (1850-1925) reformulated Maxwell's equations in their more modern vector calculus form in the late 1800s.
8. Heinrich Rudolph Herz (1857-1894) produced the transverse electromagnetic waves predicted by Maxwell's theory in the laboratory in 1888, and studied their wave properties (refraction, reflection, ...) This placed Maxwell's theory on firm experimental footing and established experimentally the connection between electromagnetism and optics suggested by Maxwell's theory.
9. *Joseph John Thomson* (1856-1940) discovered the electron in 1897. This was the first step in elaborating how electromagnetic waves interacted with matter at a microscopic level, though a full microscopic theory of atoms and molecules awaited the invention of quantum mechanics in ∼1925-1926. This, for example, allowed the classical Drude model for conduction electrons described in Section 7.3.2 to be constructed, even though there wasn't yet (in the year 1900) a theory of atoms in any modern form.
10. *Albert Einstein* (1879-1955) published his *special theory of relativity* in 1905. It was inspired by Maxwell's theory, since Einstein was strongly motivated by a desire to unify particle motion and electromagnetism, which he realized could be accomplished only if both were described by Lorentz-invariant theories.

One should add to this list the *invention of quantum mechanics* in 1925-1926, which provided a firm basis for describing electromagnetism interacting with matter. But this book is about *classical electromagnetism,* and the quantum revolution that shook the very foundations of microscopic physics beginning in 1925 is primarily part of a *quantum theory of electromagnetism*, which merits a separate book and is necessarily beyond our present scope. Nevertheless, we shall often need to allude to the microscopic behavior atoms and molecules in our discussion of electromagnetism in the presence of matter, which ultimately requires quantum mechanics. Thus, we shall freely invoke quantum mechanics, when needed, but those discussions will tend to be qualitative.

1.2 The Maxwell Equations

Thus, by the late 1800s the basics of classical electromagnetic theory could be summarized concisely in the *Maxwell equations,*which may be written in free space as

$$\boldsymbol{\nabla} \boldsymbol{E} = \frac{\rho}{\epsilon_0} \qquad \text{(Gauss's law)}, \tag{1.1a}$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \text{(Faraday's law)}, \tag{1.1b}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \qquad \text{(No magnetic charges)}, \tag{1.1c}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \mu_0 \boldsymbol{J} \qquad \text{(Ampère's law, as modified by Maxwell)}, \tag{1.1d}$$

using modern notation and SI units (see the discussion of units in Section 2.2), where

1. $\boldsymbol{E}$ is the electric field,

2. $\boldsymbol{B}$ is the magnetic field,
3. ρ is the charge density,
4. $\boldsymbol{J}$ the current vector,
5. ϵ_0 is the *permittivity of free space* [defined for SI units in Eq. (2.3)], and
6. μ_0 is the *permeability of free space,* which takes the value

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}}, \tag{1.2}$$

when expressed in SI units, where N is newtons and A is amperes. It should be noted that the SI-system constants ϵ_0 and μ_0 are *not independent* but are constrained by $\mu_0\epsilon_0 = 1/c^2$, where c is the speed of light.[1]

Maxwell's original formulation of what we now call the Maxwell equations was in terms of the vector and scalar potentials instead of the electric and magnetic fields (see Sections 2.7 and 8.6 for the relationship between the potentials and the fields), required 20 equations instead of four, and did not use the modern vector calculus notation of Eqs. (1.1), which was invented later by Oliver Heaviside and independently by J. Willard Gibbs. Heaviside reduced Maxwell's 20 equations to the four in Eqs. (1.1), by writing them entirely in terms of the electric and magnetic fields in the new vector calculus notation. One cannot overstate the importance of compact, concise, and evocative notation in the advancement of mathematical physics, and Ref. [29] may be consulted for the history of this important evolution in notation for electromagnetic equations.

An important property of the Maxwell equations is that they obey a set of symmetries, which are summarized in Box 1.1. In Ch. 12 we shall elaborate on the relationship of these symmetries to the conservation laws obeyed by classical electromagnetism.

1.3 Charge Conservation

The charge density ρ and the current vector $\boldsymbol{J}$ appearing in the Maxwell equations are not independent but are constrained by the *continuity equation,*

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{J} = 0 \qquad \text{(Continuity equation)}, \tag{1.3}$$

which ensures *conservation of charge* by requiring that variation of electrical charge in some arbitrary volume is caused by flow of electrical current through the surface of that volume. The local conservation of charge implied by Eq. (1.3) is not a separate feature, but is implied by the Maxwell equations themselves (see Problem 1.1). As indicated in Box 1.1, this conservation of charge by the equations of electromagnetism is associated with the *gauge symmetry of electromagnetism*, which will enter our later discussion in a number of forms.

[1] This relationship between ϵ_0 and μ_0 is a consequence of the units chosen for the electric field and magnetic field in the SI system.

Box 1.1 Symmetries of the Maxwell Equations

Symmetry plays a fundamental role in physics [27]. The Maxwell equations (1.1) exhibit some symmetries that have important implications for electromagnetism.

1. *Invariance under Space and Time Translations:* Invariance under translations in space and time is associated with conservation of momentum and energy, respectively. Satisfaction of these invariances by the Maxwell equations implies that *energy and momentum can be assigned to the electromagnetic field*.
2. *Invariance under Rotations:* The formulation of the Maxwell equations as vector equations ensures rotational invariance (isotropy of space), which in turn implies conservation of angular momentum by the electromagnetic field.

> The association of invariance under space translations, time translations, and rotations with conservation of linear momentum, energy, and angular momentum, respectively, is a general consequence of **Noether's theorem:** *For every continuous symmetry of a field theory Lagrangian there is a corresponding conserved quantity.* See Section 16.2 of Ref. [27].

3. *Symmetry under Space Inversion (Parity) P:* In the presence of rotational invariance, parity is equivalent to mirror reflection. Under inversion $\boldsymbol{E} \to -\boldsymbol{E}$, which is the transformation law for a *polar vector* (normal 3-vector), but under inversion $\boldsymbol{B} \to \boldsymbol{B}$, which is the transformation law for a *pseudovector* or *axial vector*.
4. *Symmetry under Time Reversal (T):* A motion picture of electromagnetic events would be consistent with the Maxwell equations if run backward or forward.
5. *Lorentz Invariance:* Electromagnetism isn't Galilean invariant but is Lorentz invariant (it is consistent with special relativity), while Newtonian mechanics is Galilean invariant but not Lorentz invariant (it is inconsistent with special relativity). The Maxwell equations (1.1) are *Lorentz covariant* (their validity is unaltered by Lorentz transformations). However, this isn't clear from the notation because space and time enter on an equal footing in special relativity and the use in Eqs. (1.1) of 3-vectors instead of 4-vectors, and of separate derivatives for space and time, obscures this covariance. Later we will re-write the Maxwell equations in a manifestly Lorentz-covariant form.
6. *Gauge Invariance:* Consistency of the Maxwell equations with Eq. (1.3) requires *charge to be conserved locally,* which implies *local gauge invariance of the electromagnetic field* (see Section 1.3). We shall have much to say about gauge symmetry and its far-reaching implications in later chapters.
7. *Electric and Magnetic Field Asymmetry:* The Maxwell equations exhibit a large *asymmetry* between electric and magnetic fields [most obvious in CGS units; see Appendix B.2]. If a magnetic charge $4\pi\rho_{\mathrm{m}}$ were added to the right side of Eq. (B.3c) and a magnetic current $(4\pi/c)\boldsymbol{J}_{\mathrm{m}}$ added to the right side of Eq. (B.3b), the Maxwell equations would become symmetric under $\boldsymbol{E} \to \boldsymbol{B}$ and $\boldsymbol{B} \to -\boldsymbol{E}$. However, *no magnetic charges have ever been observed.*

1.4 Maxwell and the Displacement Current

A feature of the Maxwell equations with profound implications is that Maxwell realized that the original form of Ampére's law [which lacked the $\partial \boldsymbol{E}/\partial t$ term of Eq. (1.1d)] was *inconsistent with Eq. (1.3)* because it was valid only for *stationary charge densities*. As will be discussed in Section 11.1.1, Maxwell then added the $\partial \boldsymbol{E}/\partial t$ term (which is called the *displacement current*) to Ampère's law in Eq. (1.1d); this brought the full set of equations (1.1) into harmony with Eq. (1.3), and effectively brought together the previously separate subjects of electricity and magnetism. As a result, Eq. (1.1d) is also called the Ampère–Maxwell law. The addition of the displacement current to Eq. (1.1d) has a number of far-reaching implications.

1. We may now speak of the unified subject of *electromagnetism*.
2. The fundamental equations of electromagnetism are now consistent with charge conservation.
3. This modification will lead eventually to the interpretation of electromagnetic waves as light waves.
4. That Maxwell's equations are consistent with the continuity equation and thus conserve charge locally will lead to the idea of classical *electromagnetic gauge invariance*, which will underlie a quantum field theory of electromagnetism (*quantum electrodynamics* or QED).
5. Electromagnetic gauge invariance will eventually be generalized to more complex gauge invariance in the weak and strong interactions, resulting in the quantum field theory that we term the *Standard Model* of elementary particle physics.

We emphasize that the introduction of the displacement current by Maxwell was not in response to any new experimental results demanding a revision of the theory; Maxwell introduced it to fix what he saw as an inconsistency in the equations thought to describe electricity and magnetism at that time. The rather remarkable consequences of the above list have been confirmed experimentally and are testament to the power of requiring mathematical consistency in physical theories.

1.5 Classical Electromagnetic Forces

The four Maxwell equations of Eqs. (1.1), supplemented by appropriate boundary conditions,[2] may be solved for the fields $\boldsymbol{E}$ and $\boldsymbol{B}$, if the charge density ρ and current $\boldsymbol{J}$ are known. However, these equations make no reference to *forces*, which are often the

[2] We will have much more to say about this later but, loosely, physically acceptable boundary conditions in static problems require fields to vanish rapidly enough at infinity, while in dynamical problems fields must represent outgoing solutions so that there is no unphysical flow of energy from infinity into regions of interest.

tangible connection to experimental results in classical physics. This is remedied by introducing the empirically justified *Lorentz force law*,

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \qquad \text{(Lorentz force)}, \tag{1.4}$$

which implies that the electromagnetic force acting on a particle having charge q and velocity $\boldsymbol{v}$ at the point $\boldsymbol{x}$ is determined completely by the instantaneous values of the fields $\boldsymbol{E}$ and $\boldsymbol{B}$ at $\boldsymbol{x}$. Then Newton's second law relating momentum change to force,

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{F}, \tag{1.5}$$

allows calculating the complete motion of charges in an classical electromagnetic field.

1.6 Internal Consistency of Electromagnetism

Classical mechanics is internally self-consistent, in the sense that there are phenomena outside its domain (very small requires quantum theory; very fast requires special relativity; very strong gravity requires general relativity), but it gives internally consistent results when restricted to its domain of validity. Classical electromagnetism is different. In Eq. (2.45) we shall show that the energy U of a continuous charge distribution $\rho(\boldsymbol{x})$ is the work W required to assemble it,

$$U = W = \frac{\epsilon_0}{2} \int E^2 d\tau,$$

where $\boldsymbol{E}$ is the electric field associated with the charge distribution and the integration is over all of space. But from Eq. (2.9) the electric field of a continuous charge distribution behaves as

$$\boldsymbol{E}(\boldsymbol{x}) = \frac{1}{4\pi\epsilon_0} \int \rho(\boldsymbol{x}')\, \frac{\boldsymbol{x} - \boldsymbol{x}'}{|\boldsymbol{x} - \boldsymbol{x}'|^3}\, d^3x',$$

and U diverges as $|\boldsymbol{x} - \boldsymbol{x}'|^{-4}$ upon approaching a point charge located at $\boldsymbol{x}'$. This *infinite self-energy* of a classical point charge may be viewed as resulting from the charge acting on itself and is clearly unphysical; the first thought is that it must be excluded from the energy of a charge distribution. But further problems arise if one considers *accelerated charges*, which radiate electromagnetic waves and must lose energy; this can be thought of most simply as a radiative damping force acting on the charged particle, which again forces us to think of the charge acting on itself, with the short-range limit diverging.

We conclude that there is no consistent way to deal with such self-energies in classical electromagnetism and the theory is internally inconsistent.[3] However, as a practical matter we may ask at what order of magnitude for distance and time scales does one expect

[3] Similar problems with infinities associated with matrix elements evaluated at a point (such as the infinite self-energies of point particles) arise in quantum field theory. There a scheme called the *renormalization program* has been developed to systematically subtract away the infinities in observable quantities and bury them in quantities that are not observable. Not everyone is comfortable philosophically with the "sweeping under the rug" nature of the quantum renormalization scheme, but it is undeniable that renormalization

such inconsistencies lead to actual problems? Consider the self-energy of an electron. As far as is known experimentally elementary particle like electrons are point-like, without spatial extent. But if the electron is assumed to have a finite radius r_e, the electrostatic energy becomes of order e^2/r_e, where e is the electron charge. The only other energy scale in the problem is the electron restmass energy $m_\mathrm{e}c^2$, and if we equate these two energy scales and solve for r_e,

$$r_\mathrm{e} = \frac{e^2}{mc^2} = 2.82 \times 10^{-13}\,\mathrm{cm}, \tag{1.6}$$

which is known as the *classical radius of the electron.* (In comparison, the Bohr radius of the hydrogen atom is 0.529×10^{-8} cm.) A corresponding timescale is given by the time for light to travel this distance,

$$\tau_e = \frac{r_\mathrm{e}}{c} = \frac{e^2}{mc^3} \simeq 10^{-21}\,\mathrm{s}. \tag{1.7}$$

Unless we consider motion of charges that changes over a length scale of order r_e, or a timescale of order τ_e, the infinities associated with self energies are of no practical significance. In fact, one expects quantum effects to become important on a length given by the *Compton wavelength* $r_\mathrm{C} = \hbar/mc$, which is

$$\frac{r_\mathrm{C}}{r_\mathrm{e}} = \frac{\hbar/mc}{e^2/mc^2} = \frac{\hbar c}{e^2} \simeq 137$$

times larger than the classical electron radius. We conclude that inconsistencies associated with self-energies of charged particles are irrelevant for our discussion.

1.7 Classical Electromagnetic Solutions

Thus the understanding of classical electromagnetism can be summarized in a succinct admonition: to be blunt, "shut up and calculate", or to be less blunt and more precise,

> *Solve Maxwell's equations with appropriate boundary conditions for the electric and magnetic fields, and use those to compute forces and observables.*

While this admonition is not wrong, it risks leaving two things at loose ends.

1. The solution of Maxwell's equations with appropriate boundary conditions can be highly non-trivial, often requiring considerable mathematical and computational prowess.
2. Although classical electromagnetism itself has changed little since the late 1800s, the context in which we view classical electromagnetism has been altered dramatically by modern advances in quantum field theory (which were often inspired and guided by the theoretical understanding of classical electrodynamics).

succeeds in removing infinities from observables, and that the resulting (finite) predictions of quantum field theory are in spectacular agreement with experimental observables.

The following chapters will address the first point practically, by providing mathematical and computational tools to facilitate solution of Maxwell's equations in various contexts. The rest of the current chapter addresses the second point more philosophically, by setting electromagnetism more broadly in the context of modern theoretical physics.

1.8 Electromagnetism in Modern Physics

Let us make some general remarks about the relationship of classical electromagnetism to modern theoretical physics.

1.8.1 Gauge Symmetry

A remarkable findings of modern physics is that the already beautiful edifice of Maxwell's equations for electromagnetism that we shall elaborate in these lectures hides within it a deceptively powerful symmetry called *(local) gauge invariance*—in essence a symmetry under local phase transformations—that was introduced in Box 1.1 and, with suitable exposition, explains the origin of both classical and quantum theories of electromagnetism. But that is not all! The local gauge symmetry describing electromagnetism can be generalized mathematically into a more powerful theory that can *partially unify* the electromagnetic and weak nuclear forces into a single *electroweak interaction*, and this can be generalized into the *Standard Model* (of elementary particle physics) that partially unifies the electromagnetic, weak, and strong interactions in a single non-abelian gauge theory.

In quantum field theory local gauge symmetries are generated by a set of quantum operators. If the quantum operators all commute among themselves, the gauge symmetry is said to be *abelian*; if they do not all commute, the gauge symmetry is said to be *non-abelian* (see the discussion of symmetries and group theory in Box 16.1). Because of the constraints arising from non-zero commutators among operators, non-abelian gauge symmetries have the potential to engender more complex behavior than abelian gauge symmetries. Electromagnetism corresponds to an abelian local gauge symmetry (with the quantum version known as *quantum electrodynamics or QED*). In Ch. 19 we shall explore in more depth this view of classical electromagnetism as an abelian gauge field theory and its relationship to quantum electrodynamics. The Standard Electroweak Model and its generalization to the Standard Model incorporating the strong interactions (*quantum chromodynamics or QCD*) generally correspond to more complex non-abelian local gauge symmetries. Non-abelian models of elementary particles are also known as *Yang–Mills field theories.*

1.8.2 Quantization of Electrical Charge

The basic unit of electrical charge is given by the magnitude of the charge on an electron, which is measured to be

$$e \equiv |q_e| = 1.60217733 \times 10^{-19}\,\mathrm{C} \qquad \text{[in SI units]}. \tag{1.8}$$

The charges on protons, and all presently known particles or systems of particles, are found to be *integral multiples of this unit* (with positive or negative signs for non-zero charges). It is known experimentally that the ratios of electrical charges between different particles are integers to one part in 10^{20}. Indeed, the stability of the atomic matter all around us would be compromised by even a tiny difference in the absolute values of the electron and proton charges, so the very existence of the observable Universe is strong evidence for quantization of electrical charge.

> Dirac proposed long ago that, if fundamental magnetic charges (magnetic monopoles) existed, there could be a topological reason for charge quantization. However, no reproducible observations of magnetic monopoles have ever been reported, so Dirac's idea remains only conjecture. As we have noted in Box 1.1, if magnetic monopoles did exist the Maxwell equations (1.1) would require modifications that would make them more symmetric with respect to electric and magnetic fields.

Since there is no evidence for the existence of free magnetic charge (magnetic monopoles), we must conclude that there is presently no convincing fundamental explanation for the observed quantization of electrical charge in integer multiples of the electron charge within classical electromagnetism. There are constraints arising from from what are called *anomalies* in relativistic quantum field theory and the phenomenology of the Standard Model of elementary particles physics outlined in Box 1.2 that might provide an explanation, but that is beyond the scope of our discussion of classical electromagnetism.

1.8.3 Charges of Known Elementary Particles

Our modern view is that matter in the Universe consists of the fermions in the Standard Model (of elementary particle physics). The elementary particles of the Standard Model are summarized in Fig. 1.1. In the Standard Model all matter is formed from fermions ($e, \nu_e, u, ...$), while interactions between elementary particles are mediated by the exchange of gauge bosons ($\gamma, G, W^\pm, Z^0$), and masses for bare (that is, non-interacting) particles arise from couplings to the Higgs boson H. For reasons that remain elusive, the fermions (matter fields) of the Standard Model may be divided into three generations (or families), I, II, and III, with the fermions of each generation being successively more massive than in the preceding generation, and with many properties repeating themselves in successive generations. To give one example, the muon μ in generation II acts in many respects as if it were a heavier version of the electron e in generation I. Table 1.1 gives

Box 1.2 Anomalies, GUTs, and Charge Quantization

In quantum gauge field theory, when a symmetry that is obeyed in a classical system is broken by the act of quantizing that system, it is termed an *anomaly*. Anomalies by themselves do not require electric charge to be quantized but for a gauge theory like the Standard Model to be renormalizable and thus self-consistent any anomalies arising in its quantization must cancel exactly, and that required cancellation effectively imposes quantization of electrical charge in order to reproduce the phenomenology of the Standard Model. In addition, the symmetry requirements in various grand unified theories (GUTs), such as those based on SU(5) or SO(10) symmetries impose quantization of charge. However, we now know that there is (as yet uncertain) physics beyond the standard model because of observed neutrino flavor oscillations (implying a finite neutrino mass), and GUTs are conjectures but not yet supported experimentally, so change quantization is suggested by anomaly cancellation in the Standard Model and by some GUTs symmetries, but cannot be taken as proven.

Table 1.1 Quantum number assignments for quarks [22]

	Up	Down	Strange	Charm	Bottom	Top
Symbol	u	d	s	c	b	t
Baryon number (B)	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
Spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
Charge (Q)	$\frac{2}{3}e$	$-\frac{1}{3}e$	$-\frac{1}{3}e$	$\frac{2}{3}e$	$-\frac{1}{3}e$	$\frac{2}{3}e$
Isospin (T)	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
Projection of isospin (T_3)	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0
Strangeness number (S)	0	0	-1	0	0	0
Charm number (c)	0	0	0	1	0	0
Bottom number (b)	0	0	0	0	-1	0
Top number (t)	0	0	0	0	0	1

The additive quantum numbers Q, T_3, S, c, B, b, and t of the corresponding antiquarks are the negative of those for quarks. The charge is given by $Q/e = T_3 + \frac{1}{2}(B + S + c + b + t)$, where e the absolute value of the electron charge.

Standard Model quark quantum number assignments for the six known quark flavors displayed in Fig. 1.1. As indicated in Table 1.1, the (electrical) charge Q of each quark is given by

$$Q/e = T_3 + \frac{1}{2}(B + S + c + b + t), \tag{1.9}$$

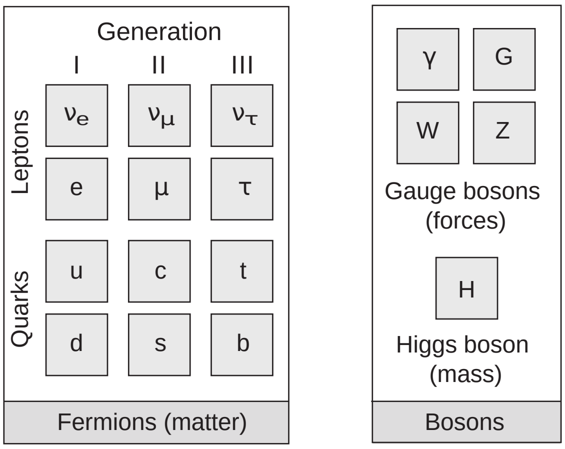

Fig. 1.1 Elementary particles of the Standard Model. Photons are labeled by γ and gluons by G. Elementary particles of half-integer spin that don't undergo strong interactions are called *leptons;* electrons and electron neutrinos are examples. Particles made from quarks, antiquarks, and gluons (and thus that undergo strong interactions) are called *hadrons*; pions (pi mesons) and protons are examples. A subset of hadrons corresponding to more massive particles containing three quarks are called *baryons*; protons and neutrons are examples. The different types of neutrinos $(\nu_e, \nu_\mu, ...)$ and the different types of quarks (u, d, s, ...) are called *flavors.* For simplicity the largely parallel classification of antiparticles has been omitted in this diagram.

which leads to third-integer charges for all the quarks (row 4 of Table 1.1).

Example 1.1 From Eq. (1.9) and Table 1.1, the charge of the down quark d is

$$\begin{aligned} Q_d/e &= T_3 + \frac{1}{2}(B + S + c + b + t) \\ &= -\frac{1}{2} + \frac{1}{2}\left(\frac{1}{3} + 0 + 0 + 0 + 0\right) \\ &= -\frac{1}{3}. \end{aligned}$$

This quantization of charge in integer multiples of $\pm\frac{1}{3}$ of the absolute value of the electron charge is characteristic of quarks, as may be seen from Table 1.1.

However, the non-abelian gauge field theory of the strong interactions (quantum chromodynamics) predicts that *quarks are confined and can never appear as free particles,*[4] and

[4] More precisely, the gauge theory of the strong interactions, quantum chromodynamics (QCD), is thought to exhibit *color confinement:* particles carrying the strong-interaction gauge charge (called whimsically "color", with no relationship to the usual meaning of color) are *confined* to the interior of the hadrons such as neu-

indeed no free particles with fractional charges have ever been observed reproducibly in experiments.

Example 1.2 A proton has a udu quark structure (two up quarks and one down quark) and charge is an additive quantum number, so the charge of the proton is the sum of the charges of its quarks. From Table 1.1, the charge of a proton is

$$Q_p = 2Q_u + Q_d = \left(\frac{2}{3} + \frac{2}{3} - \frac{1}{3}\right) e = +1e,$$

in terms of the fundamental charge unit e given by Eq. (1.8), while a neutron has a udd quark structure and

$$Q_n = Q_u + 2Q_d = \left(\frac{2}{3} - \frac{1}{3} - \frac{1}{3}\right) e = 0\,e,$$

so neutrons have zero net electrical charge.

Thus both neutrons and protons carry integer charges, even though their constituent quarks have fractional charges.

> Modern high energy physics proposes a variety of elementary particles, some of which have electrical charges that are fractions of the fundamental unit (1.8) set by the charge on the electron. However, the *physically observable particles* entering into classical electromagnetism (and into relativistic quantum field theory) all appear to have electrical charges that are integer multiples of the fundamental charge unit given by Eq. (1.8). Even if one admits quarks (not directly observable because of confinement) to the discussion, the electric charge still appears in quantized form, but in third-integer units.

Hence we shall develop the theory of classical electromagnetism assuming that charged particles carry an electrical charge quantized in integer units of the electron charge, even though we presently have scant fundamental explanation for why this should be so.

Background and Further Reading

The history of classical electromagnetism is summarized in various parts of Jackson [30]. A view of classical electromagnetism with emphasis on the relationship of modern quantum field theory to classical electromagnetism may be found in Wald [56]. Introductions

trons and protons, and can never appear as free particles. Finding solutions for the equations of QCD is extremely difficult because it is a strongly interacting, highly non-linear field theory in the limit that would lead to confinement. Thus it is not simple to prove that QCD is color-confining. However, modern large-scale computer simulations of QCD (a discipline called *lattice gauge theory*) have increasingly converged on the conclusion that QCD exhibits confinement of particles like quarks carrying a color charge.

to the Standard Model of elementary particle physics and the origin of the particle and quantum number assignments in Fig. 1.1 and Table 1.1 may be found in many places; for example, in Refs. [22, 27].