

Astronomy 411 midterm test (Fall, 2025) modified for AI readability

1. The Standard Solar Model predicts that at a solar radial distance of $R = 0.00650$ times the solar radius $R_{\odot}$, the temperature T is 1.6×10^7 K and the pressure P is 2.34×10^{17} dynes/cm². Assuming that ionization of hydrogen in the Sun is purely thermal, what is the fraction of hydrogen ionization at the center of the Sun? In reality, the Sun is 100 percent ionized in the deep interior. If your result differs from that, explain.
2. Repeat the derivation of the virial theorem in Ch. 4 for a star surrounded by a thin atmosphere so that the pressure at the surface P_0 is not zero.
3. Derive a formula for the buoyancy acceleration a_b of a localized packet of gas with uniform density ρ immersed in a larger volume of the same gas having uniform density ρ' . What is the buoyancy acceleration of a layer of air at a temperature of 30°C if the surrounding air has a temperature of 28°C ? Neglecting complications like changing parameters, friction, . . . , how fast and in what direction would this packet of air be moving because of the buoyancy acceleration after 10 minutes, if it started from rest?
4. Consider the burning of pure ^{28}Si as illustrated in Fig. 6.10. Given that the rate for the reaction $\alpha + ^{28}\text{Si} \rightarrow ^{32}\text{S} + \gamma$ is $20 \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-1}$ and that the rate for the reaction $\gamma + ^{28}\text{Si} \rightarrow \alpha + ^{24}\text{Mg}$ is $8 \times 10^{-5} \text{ s}^{-1}$, to what abundance does the α -particle population have to grow before the rate of α -capture on silicon, $\alpha + ^{28}\text{Si} \rightarrow ^{32}\text{S} + \gamma$, becomes equal to the rate for photodisintegration of silicon, $\gamma + ^{28}\text{Si} \rightarrow \alpha + ^{24}\text{Mg}$, assuming a density of 10^6 g cm^{-3} and temperature of $T_9 = 3$? Hint: Simplify by neglecting all reactions except the two listed above that change the ^{28}Si abundance directly. Note that the rates for alpha-capture and photodisintegration given above are in different units.
5. The formalism that we have developed for convection in stars can be applied with suitable modification to the atmosphere of a planet. Assume Mars to have a pure CO_2 atmosphere, a surface temperature $T = 220$ K, and a surface atmospheric density $\rho = 0.02 \text{ kg m}^{-3}$. Calculate the gravitational acceleration and atmospheric pressure at the surface, and the scale height of the atmosphere evaluated at the surface.